Birzeit University
Mathematics Department
Math 132
Final Exam
First Summer Semester 2012/2013

Student Name: ............................................ Student Number: ........................
Time: 150 minutes — There are 4 questions in 10 pages

Question 1. (60%) Circle the most correct answer:

(1) The volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $x = 1$, and the $x$-axis, about the $y$-axis, is:

(a) $\frac{3\pi}{5}$
(b) $\frac{\pi}{5}$
(c) $\frac{2\pi}{5}$
(d) $\frac{4\pi}{5}$

(2) $\sum_{n=2}^{\infty}(0.5)^{-n} =$

(a) 2
(b) 1
(c) $\frac{1}{2}$
(d) None of the above

(3) $\int_0^{\frac{\pi}{2}} \tan x \, dx =$

(a) 0
(b) $-1$
(c) $\infty$
(d) $-\infty$

(4) If $y$ is the solution of the differential equation $\frac{dy}{dx} = 3x^2y + y$, $y(1) = e$, then $y(-1) =$

(a) $-1$
(b) $-3$
(c) $e^{-1}$
(d) $e^{-3}$

(5) $\displaystyle\int_1^4 \frac{3^{\sqrt{x}}}{2\sqrt{x}}\,dx =$

(a) $\dfrac{6}{\ln 3}$

(b) $\dfrac{3}{\ln 3}$

(c) $\dfrac{78}{\ln 3}$

(d) $\dfrac{9}{\ln 3}$

(6) The volume of the solid whose base is the region enclosed between the curves $y = x^2$ and $y = x$, and whose cross sections perpendicular to the $x$-axis are equilateral triangles of height 4, is:

(a) $\dfrac{1}{2}$

(b) $\dfrac{1}{3}$

(c) $\dfrac{1}{6}$

(d) $\dfrac{1}{4}$

(7) If $a_n = n3^{\frac{1}{n}}$, $n \in \mathbb{N}$, then $\lim\limits_{n\to\infty} a_n =$

(a) 1

(b) 0

(c) $\infty$

(d) $\ln 3$

(8) $\displaystyle\sum_{n=2}^{\infty} \frac{2n-1}{n^2(n-1)^2} =$

(a) $-1$

(b) 1

(c) $\dfrac{1}{4}$

(d) 2

(9) Assuming its convergence, find the limit of the following recursively defined sequence, $a_1 = 8$, $a_{n+1} = \sqrt{a_n + 8} - 2$:

(a) 1

(b) $-4$

(c) $-2$

(d) 8

(10) $\int e^{\sqrt{2x+1}}\,dx =$

(a) $2\sqrt{2x+1}\,e^{\sqrt{2x+1}} + C$

(b) $\dfrac{e^{\sqrt{2x+1}}}{2\sqrt{2x+1}} + C$

(c) $\sqrt{2x+1}\,e^{\sqrt{2x+1}} - e^{\sqrt{2x+1}} + C$

(d) $\sqrt{2x+1}\,e^{\sqrt{2x+1}} - \sqrt{2x+1} + C$

(11) If $\tanh x = \dfrac{1}{2}$, $x < 0$, then $\operatorname{sech} x =$

(a) $\dfrac{\sqrt{5}}{2}$

(b) $\dfrac{-\sqrt{5}}{2}$

(c) $\dfrac{\sqrt{3}}{2}$

(d) $\dfrac{-\sqrt{3}}{2}$

(12) Which one of the following functions is the fastest growing as $x \to \infty$:

(a) $e^{\frac{x}{2}}$

(b) $\ln(\ln x)$

(c) $3^x$

(d) $4 + 2^x$

(13) The series $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{5^n + 2^n}$:

(a) Converges by direct comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{4^n}$

(b) Converges by direct comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}$

(c) Converges by direct comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{7^n}$

(d) Converges by summing its terms as a geometric series

(14) The series $\displaystyle\sum_{n=2}^{\infty} \frac{(n+1)\ln n}{\sqrt{n}}$:

(a) Converges by the integral test

(b) Converges by direct comparison with $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2}$

(c) Diverges by the ratio test

(d) Diverges by the $n$th-term test

(15) If $a_n = \left(1 - \frac{2}{n}\right)^{\frac{n}{2}}$, $n \in \mathbb{N}$, then $\lim\limits_{n\to\infty} a_n =$

(a) $e^{-2}$
(b) $e^{-1}$
(c) $e^{-4}$
(d) $e^{\frac{-1}{2}}$

(16) The series $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n(n+2)(n+3)}}$:

(a) Converges by direct comparison with $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^4}}$

(b) Converges by limit comparison with $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

(c) Converges by direct comparison with $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$

(d) Diverges by the ratio test

(17) $i^{215} =$

(a) $i$
(b) $-i$
(c) $1$
(d) $-1$

(18) The integral $\int\limits_{2}^{\infty} \frac{dx}{\sqrt[3]{x}\,(\sqrt{x}-1)}$:

(a) Converges by limit comparison with $\int\limits_{2}^{\infty} \frac{dx}{\sqrt[3]{x}}$

(b) Converges by limit comparison with $\int\limits_{2}^{\infty} \frac{dx}{\sqrt{x}}$

(c) Diverges by direct comparison with $\int\limits_{2}^{\infty} \frac{dx}{\sqrt[3]{x^2}}$

(d) Diverges by direct comparison with $\int\limits_{2}^{\infty} \frac{dx}{\sqrt[6]{x^5}}$

(19) The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{e^n(x-1)^n}{n^2 3^n}$ is:

(a) $\frac{3}{e}+1$

(b) $\frac{e}{3}+1$

(c) $\frac{3}{e}$

(d) $\frac{e}{3}$

(20) $\int_0^1 x^2 \ln x \, dx =$

(a) $\frac{-1}{4}$

(b) $\frac{-1}{9}$

(c) $\infty$

(d) $-\infty$

(21) The series $\sum_{n=1}^{\infty} \frac{(-1)^n (\sqrt{n}+n)}{\sqrt{n^5+1}}$:

(a) Converges absolutely

(b) Converges conditionally

(c) Diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

(d) Diverges by the $n$th-term test

(22) $\int_1^{\sqrt{3}} \frac{dx}{x\sqrt{x^2+1}} =$

(a) $\ln\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)$

(b) $\ln\left(\frac{\sqrt{2}}{\sqrt{3}+1}\right)$

(c) $\ln\left(\frac{\sqrt{2}+1}{\sqrt{3}}\right)$

(d) $\ln\left(\frac{\sqrt{3}}{\sqrt{2}+1}\right)$

(23) $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}} \sec^2 x \tan^2 x \, dx =$

(a) $\dfrac{-26}{9\sqrt{3}}$

(b) $\dfrac{28}{9\sqrt{3}}$

(c) $\dfrac{-13}{3\sqrt{3}}$

(d) $\dfrac{20}{3\sqrt{3}}$

(24) A partial fraction for the function $f(x) = \dfrac{3x+1}{x^3-8}$ is:

(a) $\dfrac{A}{x-2} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$

(b) $\dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{x+2}$

(c) $\dfrac{A}{x-2} + \dfrac{Bx+C}{x^2-2x+4}$

(d) $\dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+2x+4}$

(25) The series $\displaystyle\sum_{n=2}^{\infty} \left(\frac{n}{n^2-1}\right)^{n^2}$:

(a) Converges by summing its terms as a telescoping series

(b) Converges by the $n$th-term test

(c) Converges by the root test

(d) Diverges by direct comparison with $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{n}}$

(26) $\dfrac{4-i}{1+i} =$

(a) $\dfrac{3}{2} - \dfrac{5}{2}i$

(b) $\dfrac{3}{2} + \dfrac{5}{2}i$

(c) $\dfrac{5}{2} - \dfrac{3}{2}i$

(d) $\dfrac{5}{2} + \dfrac{3}{2}i$

(27) The series $\sum_{n=3}^{\infty} \frac{\sqrt{\ln n}}{n^{1.1}}$:

(a) Converges by limit comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{1.05}}$

(b) Converges by limit comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{1.1}}$

(c) Converges by direct comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{0.95}}$

(d) Diverges by the ratio test

(28) If $x = \ln(\sec t + \tan t)$, $y = t \sec t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$, then $\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{4}} =$

(a) $\frac{\pi}{2} + 1$

(b) $\frac{\pi}{2\sqrt{2}} + \frac{1}{\sqrt{2}}$

(c) $\frac{\pi}{4} + 1$

(d) None of the above

(29) The series $\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n+1} - \sqrt{n-1}\right)$:

(a) Converges absolutely

(b) Converges conditionally

(c) Diverges by the $n$th-term test

(d) Diverges by direct comparison with $\sum_{n=1}^{\infty} (-1)^n \sqrt{n}$

(30) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)2^{n+1}} =$

(a) $\ln\left(\frac{2}{3}\right)$

(b) $\ln\left(\frac{1}{3}\right)$

(c) $\ln\left(\frac{3}{2}\right)$

(d) $\ln\left(\frac{3}{4}\right)$

Question 2. (15%) (a) Use the binomial series to find out the first four nonzero terms of the Maclaurin series of $(1+x)^{\frac{2}{3}}$, $-1 < x < 1$.

(b) (1) Find the Taylor series of $f(x) = \tan^{-1}(3x^2)$, about $a = 0$, and specify its interval of convergence.

(2) Use the above series to estimate the value of $\tan^{-1}\left(\frac{1}{3}\right)$ with an error of magnitude less than 0.001.

Question 3. (13%) (a) Find the length of the parametric curve:

$$x = t, \quad y = \frac{t^2}{2}, \quad 0 \leq t \leq 1.$$

(b) Sketch the parametric curve defined by the equations:

$$x = 3\cos t, \quad y = 2\sin t, \quad -\frac{\pi}{2} \leq t \leq \pi.$$

Question 4. (12%) (a) Find the four forth roots of $-81$.

(b) Solve the equation: $2|z-1-i| = |z+\overline{z}-2|$.

BIRZEIT UNIVERSITY

MATH 132 Final exam 09/10

NO:

Name (........................ ....................)

INSTRUCTORS : RASEM + SHADI

## PROBLEM 1:10 MARKS

**Find the four fourth roots of** $-8+8\sqrt{3}\,i$

2

The modulus $= \sqrt{(-8)^2+(8\sqrt{3})^2} = 16$ 64

2

$\cos\theta = \frac{-8}{16} = -\frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{-8}{16}\right) = 120 = \boxed{\frac{2\pi}{3}}$ 2

$w_k = 16^{1/4}\left[\cos\left(\frac{120+2k180}{4}\right) + i\sin\left(\frac{120+2k180}{4}\right)\right]$ $k=0,1,2,3$

$w_0 = 2\left[\cos(30 + i\sin 30\right] = 2\left[\frac{\sqrt{3}}{2}+\frac{1}{2}i\right] = \sqrt{3}+i$ 1

$= 2\left[\cos(120) + i\sin(120)\right] = 2\left[-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right] = -1+i\sqrt{3}$ 1

$w_1 = 2\left[\cos(120) + i\sin(120)\right]$

$= 2\left[\cos(210) + i\sin(210)\right] = 2\left[-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right] = -\sqrt{3}-i$ 1

$w_2 = 2\left[\cos(210)\right.$

$= 2\left[\frac{1}{2} - i\frac{\sqrt{3}}{2}\right] = 1 - i\sqrt{3}$ 1

$w_3 = 2\left[\cos 300 + i\sin 300\right]$

$w_0 = \sqrt{3}+i$

$w_1 = -1+i\sqrt{3}$

$w_2 = -\sqrt{3}-i$

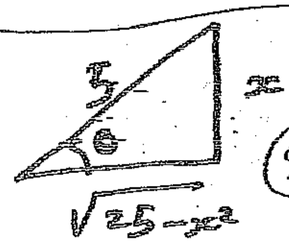
$w_3 = 1-i\sqrt{3}$

**PROBLEM 2:10 MARKS EVALUATE**

$$\int \frac{1}{x^2\sqrt{25-x^2}}dx$$

$x = 5\sin\theta$ (2)

$dx = 5\cos\theta\, d\theta$




(2)

$$\int \frac{5\cos\theta\, d\theta}{25\sin^2\theta\sqrt{25-25\sin^2\theta}}$$ (2)

$$\int \frac{5\cos\theta\, d\theta}{25\sin^2\theta(5\cos\theta)}$$ (2)

$$\frac{1}{25}\int \csc^2\theta\, d\theta$$

$$= -\frac{1}{25}\cot(\theta) + C$$ (2)

$$= -\frac{1}{25}\frac{\sqrt{25-x^2}}{x} + C$$

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**PROBLEM 3(10 MARKS):COSIDER S IS CONSTANT EVALUATE**

$$\int_0^{\infty} t^2 e^{-st}\, dt$$

$$\begin{array}{ccc} t^2 & & e^{-st} \\ & + & \\ 2t & & -\frac{e^{-st}}{s} \\ & - & \\ 2 & & \frac{e^{-st}}{s^2} \\ & + & \\ 0 & & -\frac{e^{-st}}{s^3} \end{array}$$

$$\lim_{B \to \infty} \int_0^{B} t^2 e^{-st}\, dt$$

$$= \lim_{B \to \infty} \left[ -\frac{t^2}{s e^{st}} - \frac{2t}{s^2 e^{st}} - \frac{2}{s^3 e^{st}} \right]_0^{B}$$

$$= \lim_{B \to \infty} \frac{-B^2}{s e^{sB}} - \frac{2B}{s^2 e^{sB}} - \frac{2}{s^3 e^{sB}} - \left(0 - 0 - \frac{2}{s^3}\right)$$

$$= \frac{2}{s^3}$$

**PROBLEM 4(10 MARKS)DISCUSS THE CONVERGENCE**

$$\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}}(x+4)^k$$

By Ratio test

$$\lim_{k\to\infty} \left| \frac{\frac{(k+1)^2}{2^{3(k+1)}}(x+4)^{k+1}}{\frac{k^2}{2^{3k}}(x+4)^k} \right| < 1$$

$$= \lim \frac{(k+1)^2}{k^2}\left(\frac{1}{2^3}\right)|x+4| < 1$$

$$= \lim \frac{1}{8}|x+4| < 1$$

$$= |x+4| < 8 \longrightarrow \boxed{R = 8}$$

$$-8 < x+4 < 8$$

$$\boxed{-12 < x < 4}$$

when $\boxed{x = -12}$ $\displaystyle\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}}(-12+4)^k = \sum \frac{k^2}{(8)^k}(8)^k = \sum_{k=0}^{\infty}(-1)^k k^2$

$$\lim_{k\to\infty} k^2 = \infty \text{ div.}$$

when $\boxed{x = 4}$ $\displaystyle\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}}(4+4)^k = \sum_{k=0}^{\infty} k^2$

$$\lim k^2 = \infty \text{ div}$$

$\therefore$ The interval of conv. $(-12, 4)$

# PROBLEM 5: 60 MARKS

Consider the test form the first column and the result form the second column

1. geometric series
2. p- series
3. telescoping series
4. the nth-term test
5. the integral test
6. alternating series test
7. the direct comparison test
8. the limit comparison test
9. the ratio test
10. the root test

a. converges absolutely
b. converges conditionally
c. diverges

**solve in details then circle the correct answer**

1. radius of conv.

$$\sum_{k=1}^{\infty}\frac{2^k(x-3)^k}{\sqrt{k+3}} =$$

a. $\frac{1}{2}$

b. $\frac{7}{2}$

c. $\frac{19}{6}$

d. $\frac{5}{2}$

$$\lim\left|\frac{2^{k+1}(x-3)^{k+1}}{\sqrt{k+4}}\cdot\frac{\sqrt{k+3}}{2^k(x-3)^k}\right| < 1$$

$$= \lim 2|x-3|\frac{\sqrt{k+3}}{\sqrt{k+4}} < 1$$

$$= 2|x-3| < 1$$

$$|x-3| < \frac{1}{2}$$

$-\frac{1}{2} < x-3 < \frac{1}{2}$

$\frac{5}{2} < x < \frac{7}{2}$

$x = \frac{5}{2}$ by conv. by alternating

$x = \frac{7}{2}$ div. by limit cmp. test

2. $\sum_{k=1}^{\infty}\frac{\sin k}{k^2+1}$

a. divergent by p- series
b. divergent by geometric series
c. converges conditionally
d. converges absolutely

$$a_k = \left|\frac{\sin k}{k^2+1}\right| \le \frac{1}{k^2+1} \le \frac{1}{k^2}$$

$\sum\frac{1}{k^2}$ conv. p-ser

$\sum a_k$ conv. by D.C.T.

3. $\sum_{k=0}^{\infty}\left(\frac{(-1)^k}{\sqrt{k+1}}\right) =$

a. divergent by the ratio test
b. divergent by the nth-root test
c. converges conditionally
d. converges absolutely

Let $b_k = \frac{1}{\sqrt{k}} = \frac{1}{k^{1/2}}$

$\lim \frac{a_k}{b_k} = \lim\frac{\sqrt{k}}{\sqrt{k+1}} = 1$

$\therefore b_k$ div. p-series

$\therefore \sum a_k$ div. by Limit C.T.

1) Let $a_k = \frac{1}{\sqrt{k+1}} > 0$

2) $f(x) = \frac{1}{\sqrt{x+1}} = (x+1)^{-1/2}$

$$f'(x) = \frac{-1}{2\sqrt{(x+1)^3}} < 0$$

decreasing series

3) $\lim a_k = 0$

$\sum(-1)^k a_k$ conv. alternating series

4. $\sum_{k=2}^{\infty}\frac{(-1)^k\sqrt{k}}{\ln k}=$

$\lim \frac{\sqrt{k}}{\ln k} = \lim \frac{\frac{1}{2\sqrt{k}}}{1/k} = \lim \frac{\sqrt{k}}{2} \neq 0$

a. divergent by the direct comparison test
b. divergent by the kth-term test
c. converges absolutely
d. converges conditionally

By k-th term test the series div.

5. $\sum_{k=1}^{\infty}\frac{\sqrt{n^2-1}}{n^3+2n^2+5}=$

Let $a_n = \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$

$b_n = \frac{1}{n^2}$

$\lim \frac{a_n}{b_n} = \frac{\sqrt{n^2-1}\; n^2}{n^3+2n^2+5}$

But $\Sigma b_n = \Sigma \frac{1}{n^2}$ Conv. p-series

So $\Sigma a_n$ Conv. Limit C.T.

a. convergent by the integral test
b divergent .by limit comparison test
c. convergent by limit comparison test
d. divergent by by direct comparison test

6. $\sum_{k=0}^{\infty}\frac{(-2)^{3k-1}}{9^k}=$ $\sum_{k=0} \frac{(-2)^{-1}\,((-2)^3)^k}{9^k} = \sum_{k=0} -\frac{1}{2}\left(\frac{-8}{9}\right)^k$

a. $\frac{-2}{9}$

b. $\frac{9}{7}$

c. $\frac{-9}{7}$

d. $\frac{-9}{34}$

$= -\frac{1}{2}\left[\frac{1}{1-(-\frac{8}{9})}\right]$

$= -\frac{1}{2}\left[\frac{1}{1+\frac{8}{9}}\right]$

$= -\frac{1}{2}\left[\frac{9}{9+8}\right] = \frac{-9}{34}$

7. $\sum_{k=1}^{\infty}\frac{e^{\frac{1}{n}}}{n^2}=$ $\int_1^{\infty}\frac{e^{1/x}}{x^2}\,dx$

$u = \frac{1}{x}$ | $x=1 \to u=1$, $x \to \infty \Rightarrow u \to 0$

$du = -\frac{1}{x^2}dx$

a. convergent by the integral test
b divergent .by limit comparison test
c. convergent by telescoping series
d. divergent by by direct comparison test

$e^{1/x}, \frac{1}{x^2}$ Cont., +ve, dec.

$-\int_1^0 e^u\,du$

$= \int_0^1 e^u\,du = e^u\Big]_0^1$

$= e - 1$

8. $\sum_{k=1}^{\infty} \dfrac{\cos(\frac{k\pi}{6})}{k\sqrt{k}} =$

$a_k \le \dfrac{|\cos k\pi|}{k\sqrt{k}} \le \dfrac{1}{k^{3/2}} = b_k$

$\Sigma b_k$ Conv. p-series $\Rightarrow \Sigma a_k$ Con by D.C.T.

a. divergent by the direct comparison test
b. divergent by the kth-term test
c. converges absolutely
d. converges conditionally

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9. $\lim_{x\to 0}(\cos 3x)^{\frac{5}{x}} =$

a. 1
b. 0
c. $\frac{1}{2}$
d. $\infty$

Let $y = (\cos 3x)^{5/x} \Rightarrow \ln y = \dfrac{5\ln(\cos 3x)}{x}$

~~$\ln y = 5\cos 3x / x$~~

$\lim_{x\to 0} \ln y = \lim \dfrac{5\ln\cos 3x}{x} = \lim \dfrac{-15\tan 3x}{1} = 0$

$\lim y = \lim e^{\ln y} = e^0 = 1$

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10. $\lim_{x\to 0}(1-2x)^{\frac{1}{x}} =$

a. does not exit
b. $e^{e-2}$
c. $e^{-2}$
d. $\frac{2}{e}$

Let $y = (1-2x)^{\frac{1}{x}}$

$\ln y = \dfrac{\ln(1-2x)}{x}$

$\lim_{x\to 0} \dfrac{\ln(1-2x)}{x}$

$\lim_{x\to 0} \dfrac{-2}{1-2x} = -2$

$\Rightarrow \lim (1-2x)^{\frac{1}{x}} = \lim e^{y} = e^{-2}$

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11. the sequence $a_n = \dfrac{\ln(2+e^n)}{3n}$

a. converges to $\frac{2}{3}$
b. converges to $\frac{1}{3}$
c. converges to 0
d. divergent sequences

$\lim \dfrac{\ln(2+e^n)}{3n}$

$\lim \dfrac{e^n}{\frac{2+e^n}{3}} = \lim \dfrac{e^n}{6+3e^n}$

$= \lim \dfrac{e^n}{3e^n} = \dfrac{1}{3}$

12. $\int \frac{2x^2-3x+2}{x^3+x}$ CAN BE INTERATED BY PARTIAL FRACTION

a. $\frac{A}{X}+\frac{BX+C}{X^2+1}$

b. $\frac{A}{X}+\frac{B}{X^2+1}$

c. $\frac{A}{X}+\frac{C}{X^2+1}+\frac{D}{X^2}$

d. $\frac{A}{X}+\frac{BX^2+C}{X^3}$

13. $\int_0^1 \tan^{-1}(x)dx$

a. 1

b. 0

c. $\frac{\pi}{4}$

d. $\frac{\pi}{4}-\frac{1}{2}\ln 2$

$u = \tan^{-1} x \qquad dv = dx$

$du = \frac{1}{1+x^2}dx \qquad v = x$

$x\tan^{-1}x - \int \frac{x}{1+x^2}$

$x\tan^{-1}x - \frac{1}{2}\mathrm{Ln}(1+x^2)\Big]_0^1$

$\tan^{-1}1 - \tan^{-1}(0) - [\frac{1}{2}\mathrm{Ln}2 - \frac{1}{2}\mathrm{Ln}1]$

$= \pi/4 - \frac{1}{2}\mathrm{Ln}2$

14. $\int_1^{e^{\frac{\pi}{2}}} \frac{\cos(\ln x)}{x}dx$

a. diverge

b. 1

c. 0

d. e

$u = \ln x$

$du = \frac{1}{x}dx$

$\int_0^{\pi/2} \cos u \, du = \sin u\Big]_0^{\pi/2} = 1$

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15. $\int_0^1 \frac{x}{1+3x}dx$

a. $\frac{1}{3} - \frac{1}{3}\mathrm{Ln}4 = \frac{1}{3}\int_0^1 \frac{3(x)+1}{1+3x}dx - \int_0^1 \frac{1}{1+3x}dx$

b. $\frac{1}{3}$

c. $\frac{1}{\sqrt{3}}\tan^{-1}\sqrt{3}$

d. ~~1/4~~ $\frac{1}{3}+\frac{2}{3}\mathrm{Ln}2$

$= \frac{1}{3}x\Big]_0^1 - \frac{1}{3}\mathrm{Ln}(1+3x)\Big]_0^1$

$= \frac{1}{3} - \frac{1}{3}\mathrm{Ln}4$

16. $y = e^{\sinh x}$

a. $y' = \cosh x$

b. $y' = \cosh x\, e^{\sinh x}$

c. $y' = \sinh x\, e^{\sinh x}$

d. $y' = e^{\cosh x}$

17. the center of the ellipse

$4x^2 + y^2 - 8x + 4y - 8 = 0$

Is

a.(2,4)

b. (4,2)

c.(1,2)

d. (1,-2)

$4(x^2 - 2x + 1) + (y^2 + 4y + 4) = 8 + 4 + 4$

$4(x-1)^2 + (y+2)^2 = 16$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$

18.the equation of the asymptotes for the hyperbola

$4x^2 - 3y^2 + 8x + 16 = 0$

a. $y = \frac{2}{\sqrt{3}}(x+1)$ and $y = -\frac{2}{\sqrt{3}}(x+1)$

b. $y = \frac{2}{\sqrt{3}}(x)$ and $y = -\frac{2}{\sqrt{3}}(x)$

c. $y - 1 = \frac{2}{\sqrt{3}}(x)$ and $y - 1 = -\frac{2}{\sqrt{3}}(x)$

d. $y = (x-1)$ and $y = -(x-1)$

$\frac{y^2}{4} - \frac{(x+1)^2}{3} = 1$

$a = 2$

$b = \sqrt{3}$

19. the focus of the parabola $2y = 1 - 2x - x^2$ is

a. (0,0)

b. $(-1, \frac{1}{2})$

c. (1,-1)

d. $(1, \frac{-1}{2})$

$(x+1)^2 = -2(y-1)$

$(x-h)^2 = 4p(y-k)$

$(h,k) = (-1,1)$

$p = -\frac{1}{2}$

Focus $= (-1, \frac{1}{2})$

19 the focus of the parabola $2y = 1 - x - x^2$ is

a. (0,0)

b. $(-1, \frac{1}{2})$

c. (1,-1)

d. $(1, \frac{-1}{2})$

$$x^2 + x - 1 = -2y$$
$$x^2 + x + \frac{1}{4} = -2y + 1 + \frac{1}{4}$$
$$(x + \tfrac{1}{2})^2 = -2(y - \tfrac{5}{8})$$

20. Consider the function $f(x)=\frac{1}{x}$ find the maximum error in using a Taylor polynomial Of order 3 centered at a=1to estimate 1.2

a. $(0.2)^4$

b. $(1.2)^3$

c. 1

d. $(\frac{1}{1.2})^3$

$$R_3 = \frac{24(x-1)^4}{4!\, C^5} \qquad 1 \le C \le 1.2$$
$$|R_3| \le (1-1.2)^4 = (.2)^4$$

21 From quastion20the infinite series represent

$$f(x)=\frac{1}{x} = \sum_{n=0}^{\infty} (1-x)^n$$

a. true

b. false

$$f(x) = \frac{1}{1-(1-x)} = \sum_{n=0}^{\infty} (1-x)^n$$

# MATH DEPARTMENT

## MATH 132 TEST THREE

**TIME: 60 Min.** **JANUARY 2008**

**NAME:** George Hannuneh
**NUMBER:** 106 15 15
**SECTION:** 1
Instructor's name: Dr. Rimon Jadon

82

28 30 10 14 82

**QUESTION ONE: (MULTIPLE CHOICE) [40 POINTS]**
**CIRCLE THE RIGHT ANSWER:**

1. Let $f(x)=\sum_{n=0}^{\infty}x^n$. The interval of convergence of the definite integral 0 to x, $\int_0^x f(t)dt$ is

$\frac{x^{n+1}}{n+1}\Big|_0 = \frac{x^{x+1}}{x+1} - x$ ?

$\frac{x^{n+1} - x(x+1)}{x+1}$

$\ln|1+x|$

(A) $x=0$ only
(B) $|x|\le 1$
(C) $-\infty < x < \infty$
(D) $-1 \le x < 1$
(E) $-1 < x < 1$

$-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}$

2. The coefficient of $x^4$ in the Maclaurin series for $f(x)=e^{-x/2}$ is

(A) -1/24
(B) 1/24
(C) 1/96
(D) -1/384
(E) 1/384

$\frac{(-1)^n (x)^n}{2^{n+1}\, n!}$ $\frac{1}{32}\,\frac{x^4}{4\times3\times2}$

$\frac{1}{e^{\frac{x}{2}}}$ $-\frac{e^{-\frac{x}{2}}}{2}$ $\frac{e^{-\frac{x}{2}}}{4}$ $\frac{-1}{2e^{\frac{x}{2}}}$ $\frac{1}{4e^{\frac{x}{2}}}$

3. Which of the following series diverges?
(A) $\Sigma 1/n^2$ conv
(B) $\Sigma 1/(n^2+n)$ conv.
(C) $\Sigma n/(n^3+1)$
(D) $\sum \frac{n}{\sqrt{(4n^2-1)}}$
(E) none of the preceding.

$\frac{n}{n\sqrt{4-\frac{1}{n^2}}}$ $\frac{1}{4}$

4. For which of the following series does the Ratio Test fail?
   (A) $\Sigma 1/n!$
   (B) $\Sigma n/2^n$
   (C) $1 + 1/2^{3/2} + 1/3^{3/2} + 1/4^{3/2} + \ldots$
   (D) $(\ln 2)/2^2 + (\ln 3)/2^3 + (\ln 4)/2^4 + \ldots$
   (E) $\Sigma n^n/n!$

5. Which of the following alternating series diverges?
   (A) $\Sigma (-1)^{n-1}/n$
   (B) $\Sigma (-1)^{n+1}(n-1)/(n+1)$
   (C) $\Sigma (-1)^{n+1}/\ln(n+1)$
   (D) $\sum \dfrac{(-1)^{n-1}}{\sqrt{n}}$
   (E) $\Sigma (-1)^{n-1} n/n^2 + 1$

6. Which of the following series converges conditionally?
   (A) $3 - 1 + 1/9 - 1/27 + \ldots$
   (B) $\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{4}} - \ldots\ldots\ldots\ldots$
   (C) $1/2^2 - 1/3^2 + 1/4^2 - \ldots$
   (D) $1 - 1.1 + 1.21 - 1.332 + \ldots$
   (E) $1/(1*2) - 1/(2*3) + 1/(3*4) - 1/(4*5) + \ldots$

7. Suppose $f(x)$ is a function with Taylor series converging to $f(x)$ for all $x \in \Re$.

   If $f(0) = 2, \quad f'(0) = 2$ and $f^{n}(0) = 3$ for $n \geq 2$ then $f(x)$=

   (A) $3e^x + 2x - 1$

   (B) $e^{3x} + 2x + 1$

   (C) $e^{3x} - x + 1$

   (D) $3e^x - x - 1$

   (E) $3e^x + 5x + 5$

8. Which of the following series converge?

(I) $\sum_{n=1}^{\infty} \frac{\ln(n^{-3})}{n^{-3}}$ (II) $\sum_{n=1}^{\infty} \frac{\ln 3}{3n}$ (III) $\sum_{n=1}^{\infty} \frac{n}{3^n}$

(A) I only (B) II only (C) III only

(D) None (E) I and III

$\frac{x^n}{n!}$

9. What is the Taylor series for $f(x) = e^x$ about $x = 1$?

(A) $\sum_{n=0}^{\infty} \frac{-(x-1)^n}{n!}$ (B) $\sum_{n=0}^{\infty} \frac{-e(x-1)^n}{n!}$ (C) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!e}$

(D) $\sum_{n=0}^{\infty} \frac{e(x-1)^n}{n!}$ (E) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$

$\frac{e(x-1)^n}{n!}$

10. Let $\{a_n\}$ be a sequence of positive real numbers such that

$$\frac{a_{n+1}}{a_n} \le \frac{n+4}{2n+1} \quad \text{for all } n. \text{ Then } \lim_{n\to\infty} a_n =$$

$\frac{1}{2}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} \le \frac{1}{2}$

$\Rightarrow \sum a_n$ converges

$\Rightarrow \lim_{n\to\infty} a_n = 0$.

(A) 0 (B) 1/2 (C) 1

(D) 2 (E) 4

**QUESTION TWO: [30 points]**

**Test the following series for convergence or divergence. State the test you are using, and verify that it applies. Determine whether the convergence is absolute or conditional.**

(a) $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^{100}}$ $\Rightarrow$ Converges by integral test

$$\int_3^{\infty} \frac{1}{n(\ln n)^{100}} = \frac{(\ln n)^{-99}}{-99}\Big|_3^{\infty}$$

$$= \frac{-1}{99(\ln n)^{99}}\Big|_3^{\infty} = \frac{-1}{\infty} + \frac{1}{99(\ln 3)^{99}} \Rightarrow \text{Integral Converges}$$

(b) $\sum_{n=0}^{\infty} \frac{n^{2n}}{(1+2n^2)^n}$ ⇒ Converges by the nth Root test abs

$$\lim_{n\to\infty} \sqrt[n]{\frac{n^{2n}}{(1+2n^2)^n}}$$

$$= \lim_{n\to\infty} \frac{n^2}{1+2n^2} = \boxed{\frac{1}{2}} < 1 \Rightarrow \text{converges}$$

(c) $\sum_{n=1}^{\infty} \frac{n^3 - \sqrt{n} + 6}{n^4 - n - 3}$ diverges by limit comparison test

$$\lim_{n\to\infty} \frac{n^3 - \sqrt{n} + 6}{n^4 - n - 3} \div \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{n^4 - n^{\frac{3}{2}} + 6n}{n^4 - n - 3}$$

$= 1$ ⇒ ~~both~~ both diverge or converg

$\frac{1}{n}$ diverges (power series with $p=1$)

⇒ both diverg

30

**QUESTION THREE: [14 points]**

Consider the power series $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}(x+3)^n$

(a) When $x = -4$ does this series converge or diverge?
(b) Determine all values for which the series converges.

$$\lim_{n\to\infty} \sqrt[n]{\frac{(-2)^n}{n^{\frac{1}{2}}}(x+3)^n}$$

$$= \frac{-2}{n^{\frac{1}{2n}}}(x+3) \quad \therefore \quad |x+3| < \frac{1}{2} \Rightarrow R = \frac{1}{2}$$

$$-\frac{1}{2} < x+3 < \frac{1}{2}$$

$$-4 < x < -2$$

when $x = -4$

$$\sum \frac{(-2)^n (-1)^n}{\sqrt{n}}$$

$$\sum \frac{2^n}{\sqrt{n}}$$

$$\lim_{n\to\infty} \frac{2^n}{\sqrt{n}} \div \frac{1}{\sqrt{n}}$$

$$\lim_{n\to\infty} 2^n = \infty$$

both diverg by L.C.T

when $x = -2$

$$\sum \frac{(-2)^n}{\sqrt{n}}$$

Converges conditionally b A.S.T

$\Rightarrow$ the series converges on the interval $(-4, -2]$

10

QUESTION FOUR: [16 points]

Consider the integral $\int x\cos(x^3)dx$.

(a) Write down the Maclaurin series for cos(x), $\cos(x^3)$, and x $\cos(x^3)$.

(b) Evaluate $\int_0^1 x\cos(x^3)dx$ as an infinite series.

$$\cos(x) = \frac{(-1)^n \; x^{2n}}{2n!}$$

$$\cos(x^3) = \frac{(-1)^n \; x^{6n}}{\cancel{26}n!}$$

Maclaurin

$$\cos(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$\cos(x^3) = 1 + \frac{x^6}{2!} + \frac{x^{12}}{4!} + \frac{x^{18}}{6!}$$

$$x\cos(x^3) = x + \frac{x^7}{2!} + \frac{x^{13}}{4!} + \frac{x^{19}}{6!} + \dots$$

$$\int_0^1 x\cos(x^3) = x + \frac{x^7}{2!} + \cancel{\frac{x^{13}}{4!}} + \cancel{\frac{x^{19}}{6!}} + \cancel{\frac{x^{25}}{8!}}$$

$$\int_0^1 x\cos(x^3) = \frac{2x + x^7}{2}\Big|_0^1 \quad \cancel{\text{where } R \le a_5}$$

$$\int_0^1 x\cos(x^3) = \boxed{\frac{3}{2}}$$

14

$$\lim_{x \to 0} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)}{\left(x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)}$$

$$\lim_{x \to 0} \frac{x\left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots\right)}{x\left(-1 + \frac{x}{2!} - \frac{x^2}{3!} - \cdots\right)} = -1$$

$(1+x^2)$

**Question 3.** *(2 points)* Express $\dfrac{1}{(1+x)^2}$ as a power series ~~and~~ find its radius of convergence.

( Hint: $\dfrac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$ )

$\cancel{\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n \Rightarrow \dfrac{1}{1-x} = \sum_{n=0}^{\infty} (-1)^n x^n}$

$\cancel{(-1)^n (nx)^{n-1}}$ $\quad \cancel{\dfrac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n}$

$\cancel{\dfrac{1}{(1+x)^2}}$

$\dfrac{d}{dx}\left(\dfrac{1}{1+x}\right) = 1 - x + x^2 - x^3 + \cdots$

$\dfrac{-1}{(1+x)^2} = -1 + 2x - 3x^2$

$\dfrac{-1}{1+x^2} = \sum_{n=1}^{\infty} (-1)^n (nx)^{n-1}$

$\cancel{\dfrac{1}{1+x^2} = \sum_{n=1}^{\infty} (nx)^{n-1}}$

by Ratio Test

$\lim_{n\to\infty} \left| \dfrac{((n+1)x)^n}{(nx)^{n-1}} \right| \Rightarrow \left| \dfrac{(n+1)^n x^n}{n^{n-1} x^{n-1}} \right|$

$|x| \lim_{n\to\infty} \dfrac{(n+1)^n}{n^{n-1}}$

**Question 4.** *(2 points)* Use series to find $\lim_{x\to 0} \dfrac{\sin x}{e^{-x} - 1}$

$\sin x = \sum_{n=0}^{\infty} x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!}$

$e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} \Rightarrow e^{-x} = \dfrac{(-1)^n (x^n)}{n!}$

$= 1 - x + \dfrac{x^2}{2!}$

$\dfrac{\left(x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \cdots\right)}{\left(1 - x + \dfrac{x^2}{2!} - 1\right)}$ $\quad \cancel{1 - \dfrac{x^2}{3!} + x^4}$

**BONUS.** *(2 points)* Find the radius of convergence of $\sum_{n=1}^{\infty} \dfrac{x^n n!}{(1)(3)(5)\ldots(2n-1)}$.

11. The Taylor polynomial of order 3 generated by $f(x) = e^{2x}$ about $a = 0$ is

a) $P_3(x) = 1 + 2x + x^2$
b) $P_3(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3$
c) $P_3(x) = 1 + x + 2x^2 + \frac{4}{3}x^3$
d) $P_3(x) = 1 + x + x^2$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{2x} = \frac{(2x)^n}{n!}$$

$$1 + 2x + \frac{4x^2}{2}$$

$$1 + 2x + 2x^2 + \frac{48x^3}{36}$$

12. The series $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$

a) Converges by ratio test
b) Diverges by ratio test
c) Converges to 4
d) Converges by root test

$$\frac{(2n+1)!}{(n+1)!(n+1)!} \cdot \frac{n!n!}{2n!}$$

$$\lim_{n\to\infty} \frac{(2n+1)}{(n+1)(n+1)}$$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

Question 2. *(4 points)* Given that $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

(a) Find the Maclaurin series of $\cos x^3$.

$$\cos x^3 = \sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}$$

2

(b) Use part (a) to estimate $\int_0^1 \cos x^3 \, dx$ with error less than 0.01

$$\int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!} = \int_0^1 1 + -\frac{x^6}{2!} + \frac{x^{12}}{4!} - \frac{x^{18}}{6!}$$

$$= x - \frac{x^7}{7(2!)} + \frac{x^{13}}{13(4!)} - \frac{x^{19}}{19(6!)}\Big]_0^1$$

6. The series $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n^5}}$

a) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^5}}$

b) Converges by direct comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^5}}$

c) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{n^2}$

d) Diverges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{n}}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$\lim_{n\to\infty} 1 = 1$, $1 \neq 0$ diverges

7. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges conditionally if

a) $0 < p < 1$

b) $0 \leq p < 1$

c) $0 < p \leq 1$

d) $0 \leq p \leq 1$

$p \leq 1$

8. The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$

a) Converges by integral test

b) Diverges by integral test

c) Converges by nth term test

d) None of the above

$\frac{5}{2} > 1$ Converges

9. The binomial series of $\frac{1}{\sqrt{1+x}}$ is

a) $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \cdots$

b) $1 + \frac{x}{2} - \frac{3x^2}{8} - \frac{5x^3}{16} + \cdots$

c) $1 - \frac{x}{2} - \frac{3x^2}{8} - \frac{5x^3}{16} - \cdots$

d) $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{x^3}{16} + \cdots$

$m = -\frac{1}{2}$ $\quad (1+x)^{-\frac{1}{2}}$ $\quad 1 + \sum_{n=1}^{\infty} \binom{m}{n} x^n$

10. The Maclaurin series generated by $x \sin x^2$ is

a) $x^3 + \frac{x^7}{3!} - \frac{x^{11}}{5!} + \cdots$

b) $x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!} - \cdots$

c) $x + \frac{x^5}{2!} - \frac{x^9}{4!} + \cdots$

d) $x - \frac{x^5}{2!} + \frac{x^9}{4!} - \cdots$

$\lim_{n\to\infty} \frac{1}{\ln n} \Rightarrow 0$

$\frac{3}{4}x^2$

Question 1. *(12 points)* Circle the best answer.

Root test =

1. The series $\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-2}\right)^n$

   a) Converges by root test

   b) Diverges by root test

   c) Converges by integral test

   d) Diverges by alternating series test

$\lim_{n\to\infty} \left(\frac{4n+3}{3n-2}\right)^{n}$

$\boxed{\frac{4}{3} > 1}$

2. If we approximate $e^x$ by $1 + x + \frac{x^2}{2!}$, then the error in estimating $e^{-1}$ is

   a) less than $\frac{1}{2}$

   b) less than $\frac{1}{2e}$

   c) less than $\frac{1}{6}$

   d) less than $\frac{1}{e}$

$\boxed{x=-1}$ $\boxed{a=0}$ $n=2$

$e^x = 1 + x + \frac{x^2}{2!}$ $e^x = \frac{x^n}{n!}$

$\left|\frac{f^{n+1}(c)(x-a)^{n+1}}{(n+1)!}\right| < \frac{M(x-a)^{n+1}}{(n+1)!}$ $a < c < x$

$0 < c < -1$

$e^c < e^{-1}$

$\left|\frac{e^c (x)^3}{3!}\right| < \frac{e^{-1}(x)^3}{3!}$

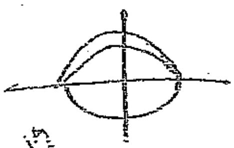
3. The radius of convergence of the series $\sum_{n=0}^{\infty} (n+1)!\,(x-4)^n$ is

   a) $R = 0$

   b) $R = 1$

   c) $R = 4$

   d) $R = \infty$



$= \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

Ratio Test

$\lim_{n\to\infty} \left|\frac{(n+1+1)!\,(x-4)^{n+1}}{(n+1)!\,(x-4)^n}\right|$

4. The series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$

   a) Converges absolutely

   b) Converges conditionally

   c) Diverges by alternating series test

   d) Diverges by nth term test

$-1 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}$

5. $1 + \pi + \frac{\pi^2}{2!} + \frac{\pi^3}{3!} + \cdots =$

   a) 0

   b) $-1$

   c) $e^\pi$

   d) None of the above

$\lim_{n\to\infty} \frac{(n+2)\cancel{(n+1)!}\,\cancel{(x-4)^n}(x-4)}{\cancel{(n+1)!}\,\cancel{(x-4)^n}}$

$|x-4|$

$\frac{1}{\sqrt{n}}$ - $a_n = \frac{1}{\sqrt{n}}$ $u_n > 0$ decreasin

$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

12/20

جامعة بيرزيت
BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT
MATH132 - ~~THIRD EXAM~~
SUMMER 2013/2014

- Name...[redacted]......
- Number.[redacted]........

- (For Question 1) Fill your answers in the tables below:

| Page 1 | | |
|---|---|---|
| 1 | b | c |
| 2 | d | c |
| 3 | d | A |
| 4 | a | B |
| 5 | C | |

| Page 2 | | |
|---|---|---|
| 6 | a | C |
| 7 | d | C |
| 8 | d | B |
| 9 | a | |
| 10 | b | |

| Page 3 | | |
|---|---|---|
| 11 | b | |
| 12 | a | B |

- Instructions:
  1. No Calculators.
  2. Mobiles Off.
  3. BZU ID On Your Desk.
  4. No Cheating At All.
  5. Time Limit: 60 Minutes.

5

First

95

Birzeit University- Mathematics Department
Calculus II-Math 132

First Exam

Spring 2012/2013

Name(Arabic):...Jineen...Jamal...Shihab..............

Number:..1120166.....

Instructor of Discussion(Arabic):..Dr....Iflefil............

Section:(10)...............12:00 الثلاثاء الأحد

**Time: 90 Minutes**

**There are 4 questions in 7 pages.**

**Question 1.**(51%) Circle the correct answer:

1. $\sinh(\ln 2) =$

   (a) $\frac{3}{2}$.

   (b) $\frac{3}{4}$.

   (c) $\frac{2}{4}$.

   (d) $\frac{5}{2}$.

$$\frac{e^x - e^{-x}}{2} = \frac{e^{\ln 2} - e^{-\ln 2}}{2} = \frac{\frac{2\times 2 - 1}{2\times 1}}{2} = \frac{4-1}{2} \cdot \frac{1}{2} = \frac{3}{4}$$

2. To solve $\int \frac{x^3+2}{x^4-1}dx$ using partial fractions, we write

   (a) $\frac{x^3+2}{x^4-1} = \frac{Ax^4+Bx^3+Cx^2+Dx+E}{x^4-1}$.

   (b) $\frac{x^3+2}{x^4-1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x^2+1}$.

   (c) $\frac{x^3+2}{x^4-1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$.

   (d) $\frac{x^3+2}{x^4-1} = \frac{Ax^2+Bx}{x^2-1} + \frac{Cx+D}{x^2+1}$.

$$\frac{x^3+2}{(x^2-1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{(x+1)} + \frac{Cx+D}{(x^2+1)}$$

3. $\int_0^1 xe^{2x}\,dx =$

   (a) $1+e^2$.

   (b) $\frac{1+e^2}{4}$.

   (c) $\frac{1+e^2}{2}$.

   (d) $\frac{e^2}{4}$.

| | |
|---|---|
| $x$ | $e^{2x}$ |
| $1$ | $\frac{e^{2x}}{2}$ |
| $0$ | $\frac{e^{2x}}{4}$ |

$u = 2x$

$du = 2\,dx$

$dx = \frac{du}{2}$

$$\frac{x}{2}e^{2x} - \frac{e^{2x}}{4}$$

$$e^{2x}\left(\frac{x}{2} - \frac{1}{4}\right)\Big]_0^1 = e^2\left(\frac{1}{2} - \frac{1}{4}\right) - \left(1\left(0 - \frac{1}{4}\right)\right)$$

$$e^2\left(\frac{1}{4}\right) + \frac{1}{4} = \frac{e^2+1}{4}$$

4. The half-life of polonium is 139 days. The decay rate $k$ is

   (a) $\frac{139}{2}$.

   (b) $\frac{2}{139}$.

   (c) $\frac{139}{\ln 2}$.

   (d) $\frac{\ln 2}{139}$.

$$t_{\frac{1}{2}} = \frac{\ln 2}{k}$$

$$\frac{\ln 2}{139}$$

5. One of the following statements is <u>false</u>

(a) $\sinh x + \cosh x = e^x$.

$\frac{e^x - \cancel{e^{-x}} + e^x + \cancel{e^{-x}}}{2}$

(b) The range of $\sinh x$ is $(-\infty, \infty)$.

(c) $\cosh 0 = 1$.

(d) $\frac{d}{dx}(\text{sech}x^2) = 2x(\tanh x)\text{sech}x$. X

$-\text{sech}\,x^2(\tanh x^2) \cdot 2x$

6. $\int_1^e \frac{2^{\ln x}}{x} dx$

(a) $\frac{1}{\ln 2}$.

(b) $\ln 2$.

(c) 2.

(d) 1.

$= \int \frac{2^u}{\cancel{x}} \cdot du\,\cancel{x}$ $\quad \int 2^u du$

$u = \ln x$

$du = \frac{1}{x} dx$

$dx = du\, x$

$= \frac{2^u}{\ln 2} \Big|$

$= \frac{2^{\ln x}}{\ln 2}\Big|_1^e = \frac{2^{\ln e}}{\ln 2} - \left(\frac{1}{\ln 2}\right) = \frac{2-1}{\ln 2} = \frac{1}{\ln 2}$

7. If $f'(x) = \tan x$, the length of the curve $f(x)$, $0 \le x \le \frac{\pi}{4}$ is

(a) $1 + \ln(\sqrt{2})$.

(b) $\ln 2$.

(c) $\ln(\sqrt{2} + 1)$.

(d) $\ln \sqrt{2}$.

$L = \int_0^{\pi/4} \sqrt{1 + (f')^2}$

$\int_0^{\pi/4} \sqrt{1 + \tan^2} = \int_0^{\pi/4} \sec x = \ln|\sec + \tan|\Big|_0^{\pi/4}$

$= \ln|\sqrt{2} + 1| - (\cancel{\ln 1 - 0})$

8. Using the substitution $x = \sin\theta$, we can write $\int \frac{\sqrt{1-x^2}}{x^2} dx$ as

$dx = \cos\theta$

(a) $\int \csc\theta \, d\theta$.

(b) $\int \cot\theta \csc\theta \, d\theta$.

(c) $\int \csc^2\theta \, d\theta$.

(d) $\int \cot^2\theta \, d\theta$.

$\int \frac{\sqrt{1-\sin^2\theta}}{\sin^2\theta} = \int \frac{\cos\theta}{\sin^2\theta} \cdot \cos\theta$

$\int \frac{\cos^2\theta}{\sin^2\theta}$

$\cancel{= \int \frac{\cos\theta \csc\theta\, d\theta}{\sin\theta}}$

$= \int \cot^2\theta$

9. The area of the surface generated by revolving the line $y = x$, $0 \le x \le 1$ about the $y$-axis is

(a) $\sqrt{2}\pi$.

(b) 2.

(c) $\sqrt{2}$.

(d) $2\pi$.

$\int 2\pi x \sqrt{1 - \left(\frac{dx}{dy}\right)^2}\, dy$ $\quad \frac{dx}{dy} = 1$

$= \int_0^1 2\pi y \sqrt{1 -}$

10. $\int_1^e \ln\sqrt{x}\,dx =$

(a) $e-1$.
(b) $\ln(1+e)$.
(c) $\frac{1}{2}$.
(d) 1.

$U = \ln\sqrt{x}$ $\quad dV = dx$

$dU = \frac{1}{2\sqrt{x}}\cdot\frac{1}{\sqrt{x}}\,dx \quad V = x$

$= x\ln\sqrt{x} - \int (x)\frac{1}{2x}\,dx = x\ln\sqrt{x} - \frac{1}{2}x \Big|_1^e$

$= \left(e\ln e^{\frac{1}{2}} - \frac{1}{2}e\right) - \left(0 - \frac{1}{2}\right)$

$= e\ln e^{\frac{1}{2}} - \frac{1}{2}e + \frac{1}{2}$

$\frac{e}{2} - \frac{e}{2} + \frac{1}{2}$

11. $\int_0^{\pi/4} \tan^3\theta\,d\theta =$

(a) $\frac{1}{2} + \ln\left(\frac{1}{\sqrt{2}}\right)$.
(b) $\frac{1}{2} + \ln\sqrt{2}$.
(c) $1 + \ln\sqrt{2}$.
(d) $\frac{\pi}{2}$.

$\int_0^{\frac{\pi}{4}} \tan\theta(\sec^2\theta - 1)$

$= \int \tan\theta\sec^2\theta - \int \tan\theta$

$u = \tan\theta$ $\quad du = \sec^2\theta$ $\quad d\theta = \frac{du}{\sec^2\theta}$

$\sec^2\theta = \tan^2\theta + 1$

$= \int u \sec^2\theta \cdot \frac{du}{\sec^2\theta}$

$\frac{u^2}{2} = \left.\boxed{\frac{\tan^2}{2}} - \ln|\cos\theta|\right|_0^{\frac{\pi}{4}}$

$= \frac{1}{2} - (\ln\frac{1}{\sqrt{2}}) - (0$

$\left(\frac{1}{2} + \ln\frac{1}{\sqrt{2}}\right)$

(1, 0)

12. A population of bacteria grows at the rate of ln 2 per hour. If the population now is 1000 bacteria, after 3 hours the population will be

(a) 3000.
(b) 8000.
(c) 4000.
(d) 6000.

$\ln 2\; e^{-k}$

$y = 1000e^{-3\ln 2}$

$3^2 \cdot 3^{-x}$

13. If $4^x = 3^{2-x}$ then $x =$

(a) $-\frac{\ln 9}{\ln 12}$.
(b) $-\frac{\ln 3}{\ln 12}$.
(c) $\frac{\ln 9}{\ln 4}$.
(d) $\frac{\ln 9}{\ln 12}$.

$x\ln 4 = 2 - x\ln 3$

$\frac{\ln 4}{\ln 3} = \frac{2-x}{x}$

$\frac{4^x}{3^{-x}} = 3^2$

$\frac{4^x}{3^{-x}} = 9$

$4^x \cdot 3^x = 9$

$(4\times 3)^x = 9$

$\ln(12)^x = \ln 9$

$\frac{x\ln 12}{\ln 12} = \frac{\ln 9}{\ln 12}$

$x\ln 12 = \ln 9$

14. The volume of the solid whose cross sections perpendicular to the $x$−axis are disks with diameters running from $y = -\sqrt{x}$ to $y = \sqrt{x}$, $0 \le x \le 1$ is

(a) $\frac{\pi}{2}$.
(b) $\pi$.
(c) $\frac{1}{2}$.
(d) $2\pi$.

$A = \pi r^2$

$\int_0^1 \pi x = \frac{\pi x^2}{2}\Big|_0^1 = \frac{\pi}{2}$

$r = \frac{\sqrt{x}+\sqrt{x}}{2} = \frac{2\sqrt{x}}{2} = \sqrt{x}$

15. The area of the surface generated by revolving the curve $y = e^x$, $0 \le x \le 1$ about the $x$−axis is

(a) $S = 2\pi \int_1^e u\sqrt{1+u^2}\,du$.

(b) $S = 2\pi \int_1^e u^2\sqrt{1+u^2}\,du$.

(c) $S = 2\pi \int_1^e \sqrt{1+u}\,du$.

(d) $S = 2\pi \int_1^e \sqrt{1+u^2}\,du$.

$S = \int 2\pi y \sqrt{1+\frac{dy}{dx}}\,dx$

$\frac{dy}{dx} = (e^x)^2 = (e^{2x})$

$S = \int_0^1 2\pi e^x \sqrt{1+e^{2x}}$

$= \int 2\pi e^x$

$u = 1 + e^{2x}$

$du = 2e^{2x}\,dx$

$dx = \frac{du}{2e^{2x}}$

$u = e^x$

$du = e^x\,dx$

$dx = \frac{du}{e^x}$

16. One of the following is true

(a) $e^x$ and $e^{2x}$ grow at the same rate.
(b) $x$ grows faster than $\ln x$.
(c) $x$ and $\ln x$ grow at the same rate.
(d) $x^{99}$ grows faster than $2^x$.

$\lim_{x\to\infty} \frac{\ln x}{x} = \frac{1}{x} = 0$

$S = \int_1^e 2\pi \sqrt{1+u^2}\,du$

17. $\int_0^1 e^x \cosh x\,dx =$

(a) $\frac{e^2-3}{4}$.
(b) $\frac{e^2+1}{4}$.
(c) $e^2+1$.
(d) $\frac{e^2}{4}$.

$e^x\left(\frac{e^x+e^{-x}}{2}\right) = \int \frac{e^{2x}+1}{2} = \frac{1}{2}\left[\frac{e^{2x}}{2}+x\right]_0^1$

$= \frac{1}{2}\left[\left(\frac{e^2}{2}+1\right) - \left(\frac{1}{2}\right)\right]$

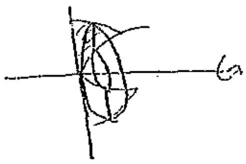
$= \frac{1}{2}\left[\frac{e^2}{2}+\frac{1}{2}\right) = \frac{e^2+1}{2}\cdot\frac{1}{2}$

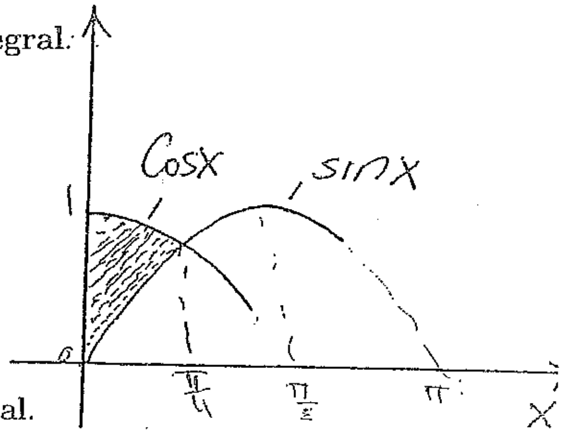
$\frac{e^2+1}{4}$

**Question 2**(16%) Consider the area enclosed between the curves $y = \sin x$, $y = \cos x$ and the $y$-axis. Setup integrals that give the volume of the solid generated by revolving this area about

(i) The $x$-axis. Use washer method. **Don't solve the integral.**

$$V = \pi \int_0^{\frac{\pi}{4}} \cos^2 x - \sin^2 x \, dx$$



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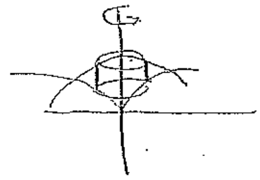
(ii) The $y$-axis. Use shell method. **Don't solve the integral.**

$$V = \int_0^{\frac{\pi}{4}} 2\pi (x) (\cos x - \sin x) \, dx$$



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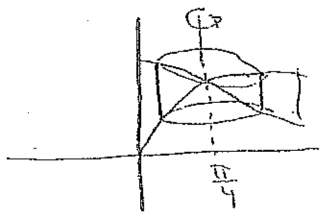
(iii) The line $x = \frac{\pi}{4}$. Use shell method. **Don't solve the integral.**

$$V = \int_0^{\frac{\pi}{4}} 2\pi \left(\frac{\pi}{4} - x\right)(x) \, dx$$

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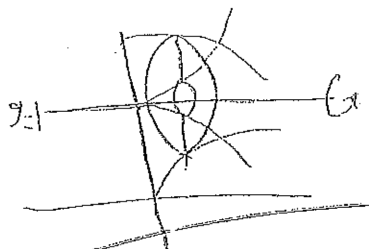
(iv) The line $y = 1$. Use washer method. **Don't solve the integral.**

$$V = \pi \int_0^{\frac{\pi}{4}} (1 - \sin x)^2 - \cos^2 x \, dx$$

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$\cos x = \sin x$

$\frac{\sin x}{\cos x} = 1$

$\frac{\pi}{4}$

14

**Question 3**(16%) Solve the following integrals:

(a) $\int \sin^{-1} x\, dx$.

$U = \sin^{-1} x \qquad dV = dx$

$dU = \frac{dx}{\sqrt{1-x^2}} \qquad V = x$

$= x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\, dx$ +8

$\int \frac{x}{\sqrt{1-x^2}}\, dx = \int \frac{\cancel{x}}{\sqrt{U}} \cdot \frac{dU}{-2\cancel{x}} = -\frac{1}{2}\int \frac{dU}{\sqrt{U}} = -\frac{1}{2}\int U^{-\frac{1}{2}}\, dU$

$U = 1 - x^2$

$dU = -2x\, dx$

$dx = \frac{dU}{-2x}$

$= -\frac{1}{2} \cdot 2 U^{\frac{1}{2}} = -\sqrt{1-x^2}$

$\int \sin^{-1} x\, dx = x\sin^{-1}x + \sqrt{1-x^2} + C$

(b) $\int \frac{(x-1)dx}{(x+1)(x^2+1)}$. $= \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

$(x-1) = A(x^2+1) + (Bx+C)(x+1)$

$(x-1) = Ax^2 + A + Bx^2 + Bx + Cx + C$

$x - 1 = (A+B)x^2 + (B+C)x + A + C$ +8

$0 = A + B \rightarrow ① \longrightarrow A = -B$

$1 = B + C \rightarrow ②$

$-1 = A + C \rightarrow ③ \longrightarrow -1 = -B + C$

$1 = B + C$

$\boxed{C = 0}$ $\boxed{B = 1}$ $\boxed{A = -1}$

$\int \frac{x-1}{(x+1)(x^2+1)}dx = \int \frac{-1}{x+1}dx + \int \frac{x}{x^2+1}dx$

$= -\ln|x+1| + \frac{1}{2}\ln|x^2+1| + C$

16

17

**Question 4**(17%) Consider the curve $y = \ln x$, $1 \le x \le \sqrt{3}$.

(a) Show that the length of the curve $L = \int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x} dx$.

$L = \int_1^{\sqrt{3}} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$\frac{dy}{dx} = \frac{1}{x} dx \rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{x^2} \rightarrow \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\frac{1}{x^2}}$

$S = \int_1^{\sqrt{3}} \sqrt{\frac{1}{1}+\frac{1}{x^2}}\, dx = \int_1^{\sqrt{3}} \sqrt{\frac{x^2+1}{x^2}}\, dx = \int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{\sqrt{x^2}}\, dx = \int_1 \frac{\sqrt{1+x^2}}{x}\, dx$

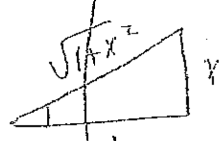
(b) Solve the integra in (a).

$\int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x}$ , $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$= \int_1^{\sqrt{3}} \frac{\sqrt{1+\tan^2\theta}}{\tan\theta} \sec^2\theta\, d\theta = \int_1^{\sqrt{3}} \frac{\sec^3\theta}{\tan\theta}\, d\theta$

~~$u = 1+x^2$, $du = 2x\,dx$, $dx = \frac{du}{2x}$~~



$\int_1^{\sqrt{3}} \sqrt{\frac{1+x^2}{x^2}} = \int_1^{\sqrt{5}} \frac{\sqrt{1+x^2}}{x} dx = \int \frac{u^2}{x^2}\, du$

$u = \sqrt{1+x^2}$

$du = \frac{2x}{2\sqrt{1+x^2}} dx$

$dx = \frac{\sqrt{1+x^2}}{x}$

$u^2 = 1+x^2$

$u^2 - 1 = x^2$

$= \int \frac{u^2}{u^2-1} du \longrightarrow u^2-1 \overline{\big) u^2}$ , $\frac{-u^2 \pm 1}{1}$

$= \int 1\, du + \int \frac{1}{u^2-1} du$

$= \frac{A}{u-1} + \frac{B}{u+1} \Rightarrow A(u+1) + B(u-1)$



$1 = Au + A + Bu - B$

$1 = (A+B)u + (A-B)$

$u = \tan\theta\, d\theta$, $du = \sec^2\theta\, d\theta$

$u = \sqrt{1+x^2}$, $du = \frac{x}{\sqrt{1+x^2}}$

$\sec \cdot \sec^2$ / $\tan\theta$

$dx = \frac{\sqrt{1+x^2}}{x} dx$, $\sqrt{1+\frac{1}{x^2}}$

$u - 1 = \frac{1}{x^2}$, $u = 1 + \frac{1}{x^2}$

$du = \frac{-1}{x} dx$

$\frac{1}{\cos\theta}$, $\frac{\cos\theta}{\sin\theta}$, $\frac{1}{\cos\theta}\ \frac{1}{\sin\theta}$

$dx = -du\, x$

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# Birzeit University
## Mathematics Department
Math 132 - First Exam
Fall 2012/2013

StudentName: [redacted] ........................ [redacted] ......Number: [redacted]

Instructors:

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**Question 1. (36%).Circle the most correct answer:**

1. $\int \frac{dx}{x(\ln x)^2} =$

   (a) $\ln(\frac{1}{x}) + c.$

   (b) $\frac{\ln x}{x} + c.$

   (c) $\frac{x}{\ln x} + c.$

   (d) $\frac{-1}{\ln x} + c.$

$\ln x = u$, $\frac{dx}{x} = du$, $dx = x\,du$ $\Rightarrow \int \frac{x\,du}{x(u)^2} \Rightarrow \int u^{-2}du$ $\quad \frac{u^{-2+1}}{-2+1}$ $\quad -\frac{1}{u}$

$= \frac{-1}{u} \Rightarrow \frac{-1}{\ln x} + c$

2. $\int \frac{2x+(x+5)^{\frac{1}{3}}}{x+5} dx =$

   (a) $2(x+5) + 10\ln|x+5| - 3\sqrt[3]{x+5} + c.$

   (b) $2(x+5) + 10e^{(x+5)} + 3\sqrt[3]{x+5} + c.$

   (c) $2(x+5) - 10\ln|x+5| + 3\sqrt[3]{x+5} + c.$

   (d) $2(x+5) - 10e^{(x+5)} - 3\sqrt[3]{x+5} + c.$

$\Rightarrow \int \frac{2x}{x+5} + \int \frac{(x+5)^{1/3}}{x+5} \Rightarrow \int (x+5)^{-2/3}$ $\Rightarrow u = x+5$, $du = 1$ $\Rightarrow \int (u)^{-2/3+1}$ $\quad 3u^{1/3}$ $\quad 3\sqrt[3]{x+5}$

$x+5 \overline{)\,2x\,}$ $\quad 2$ $\quad \frac{2x+10}{-10}$

$\Rightarrow \int 2 + \frac{-10}{x+5}dx$

$\Rightarrow 2x + -10\ln|x+5| + 3\sqrt[3]{x+5}$

3. If $\cosh x = \frac{5}{4}$, $x < 0$ then $\sinh x$

   (a) $\frac{3}{4}$.

   (b) $\frac{-4}{3}$.

   (c) $\frac{4}{3}$.

   (d) $\frac{-3}{4}$.

$\cosh^2 x - \sinh^2 x = 1$

$\frac{25}{16} - \sinh^2 x = 1$

$\frac{25}{16} - \frac{16}{16} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

4. $\int_0^1 \frac{dx}{x^5 - 1}$ is

   (a) converges.

   (b) diverges.

$\frac{1}{x^5-1} > \frac{1}{x^5}$ $\quad \int x^{-5} = -\frac{x^{-4}}{4}$

$\frac{1}{x^5}$ $\quad 0 < p < 1 \Rightarrow \infty$

$\frac{\frac{1}{x^5}}{\frac{1}{x^5-1}}$

9. $\int x^2 \tan^{-1}(2x)dx =$

(a) $\frac{x^3}{3}\tan^{-1}(2x) - \frac{1}{48}\ln(1+4x^2) + \frac{x^2}{12} + c.$

(b) $\frac{x^3}{3}\tan^{-1}(2x) + \frac{1}{48}\ln(1+4x^2) + \frac{x^2}{12} + c.$

(c) $\frac{x^3}{3}\tan^{-1}(2x) + \frac{1}{48}\ln(1+4x^2) - \frac{x^2}{12} + c.$

(d) $\frac{x^3}{3}\tan^{-1}(2x) - \frac{1}{48}\ln(1+4x^2) - \frac{x^2}{12} + c.$

10. $\int \sin x \cos^2(2x)dx =$

(a) $\frac{4}{3}\cos^3 x - \frac{4}{5}\cos^5 x - \cos x + c.$

(b) $\frac{4}{3}\sin^3 x - \frac{4}{5}\sin^5 x - \sin x + c.$

(c) $\frac{4}{3}\sin^3 x - \frac{4}{5}\cos^5 x + \sin x + c.$

(d) $\frac{4}{3}\cos^3 x - \frac{4}{5}\sin^5 x + \cos x + c.$

11. $\int \frac{dx}{(1-9x^2)^{(\frac{3}{2})}}$

(a) $\frac{x}{\sqrt{1-9x^2}} + c.$

(b) $\frac{x}{3\sqrt{1-9x^2}} + c.$

(c) $\frac{3x}{\sqrt{1-9x^2}} + c.$

(d) $\frac{\sqrt{1-9x^2}}{3x} + c.$

12. Solve for $x$: $4^{(\log_2 x)} - 3e^{(\ln x)} = 10^{(\log 4)}$

(a) $x = 4,\ x = -1.$

(b) $x = -1.$

(c) $x = -4,\ x = 1.$

(d) $x = 4.$

$\int \sin x(2\cos^2 x - 1)(2\cos^2 x - 1)$

$u = \cos x$

$du = -\sin x$

$\Rightarrow -\int 4u^4 - 4u^2 + 1$

$\Rightarrow -\frac{4u^5}{5} + \frac{4u^3}{3} - u$

$\Rightarrow -\frac{4\cos^5 x}{5} + \frac{4\cos^3 x}{3} - \cos x$

$\frac{dx}{(1-9x^2)^{3/2}}$

$x^2 - 3x = 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1)$

Question 2. *(10%) Find the length of the curve* $x = (\frac{y}{4})^2 - 2\ln\frac{y}{4}$, $4 \le y \le 12$. *(DO NOT EVALUATE THE INTEGRAL)*

$x' = \frac{y}{8} - \frac{2}{y}$

$x'^2 = \left(\frac{y}{8} - \frac{2}{y}\right)^2$

$x'^2 + 1 = \left(\frac{y}{8} - \frac{2}{y}\right)^2 + 1$

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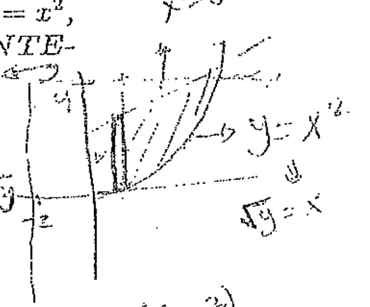
$L = \int \sqrt{\left(\frac{y}{8} - \frac{2}{y}\right)^2 + 1}\; dy$

Question 3. *(10%) Find the volume generated by revolving the area between* $y = x^2$, $x = y - 2$, *in the first quadrant about. (DO NOT EVALUATE THE INTEGRALS)*



$x = y-2 \Rightarrow y = x+2$

$y = x^2$, $\sqrt{y} = x$

1. $x = -2$ *using Shell method*

$V = 2\pi \int (\text{shell radius})(\text{shell height}) \Rightarrow R = \sqrt{y}+2$

5 $V = 2\pi \int (\sqrt{y}+2)(y-2)\sqrt{y}\; dy$

2. $y = 4$ *using Washer method*

$V = \pi \int R^2 - r^2 \quad R = 4 - x^2, \; r = 4-(x+2) = 2-x$

2 $\Rightarrow V = \pi \int (4-x^2)^2(2-x)^2 dx = \pi \int 2 - x^2 + x\; dx.$

$(y - x^2) = y - x^2 - 2 + x$

$-\frac{1}{2x^2}$

Question 4. *(16%) Determine wether the following integrals converge or diverge:*

(a) $\int_0^\infty \frac{dx}{\sqrt{x^6+1}}$

6 $\Rightarrow \int \frac{dx}{\sqrt{x^6+1}} < \int \frac{dx}{x^3}$

$\lim_{x\to 0^+} \frac{-1}{2x^2} = -\infty \quad \lim_{x\to\infty} \frac{-1}{2x^2} = 0$

$\lim_{x\to 1} \frac{-1}{2x^2} = -\frac{1}{2}$

$\Rightarrow \int_0^1 \frac{dx}{\sqrt{x^6+1}} + \int_1^\infty \frac{dx}{\sqrt{x^6+1}} \rightarrow$ by D.C.T with $\frac{1}{x^3}$ $\int_1^\infty \frac{dx}{\sqrt{x^6+1}}$ is converge

$\lim_{x\to\infty} \frac{\frac{1}{x^3}}{\frac{1}{\sqrt{x^6+1}}} = \lim_{x\to\infty}$

$\int_0^1 \frac{1}{x^p}\; 0<p<1 \Rightarrow \infty$ therefore $\int_0^1 \frac{dx}{\sqrt{x^6+1}} \rightarrow \infty$

So $\int_0^\infty \frac{dx}{\sqrt{x^6+1}}$ is diverge.

(b) $\int_0^\infty \frac{dx}{x^3 + x^{\frac{2}{3}}}$

$\frac{1}{\sqrt{3}}$

5 $\int_0^1 \frac{dx}{x^3+x^{2/3}} + \int_1^\infty \frac{dx}{x^3+x^{2/3}}$

by D.C.T with $\frac{1}{x^3}$ $\int_1^\infty \frac{dx}{x^3+x^{2/3}}$ is converge

$\int_0^1 \frac{1}{x^p}\; 0<p<1$

$\Rightarrow$ diverge, Therefore $\int_0^1 \frac{dx}{x^3+x^{2/3}}$ is diverge $\Rightarrow \int_0^\infty \frac{dx}{x^3+x^{2/3}}$ is diverge.

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5. If $f(x) = \sinh x$, $g(x) = e^x$, then as x approaches infinity

(a) $f(x)$ grows faster than $g(x)$.

(b) $f(x)$ grows slower than $g(x)$.

(c) $f(x)$ and $g(x)$ grow at the same rate.

(d) none of the above.

$$\frac{\frac{e^x - e^{-x}}{2}}{e^x} = \frac{1}{2}\left(\frac{e^x - e^{-x}}{e^x}\right)$$

6. $\int_0^{\ln 2} e^{(2x)} \cosh x dx =$

(a) $\frac{2}{3}$.

(b) $\frac{5}{3}$.

(c) $\frac{3}{5}$.

(d) $\frac{3}{2}$.

$$e^{(2x)}\left(\frac{e^x + e^{-x}}{2}\right)$$

$$\frac{1}{2}\left(\frac{e^{3x}}{3} + e^x\right)\Big|_0^{\ln 2}$$

7. The volume of the solid whose cross sections are circular disks whose diameters run from $y = x^2$ to $y = x$, $0 \le x \le 1$

(a) $\frac{\pi}{120}$.

(b) $\frac{\pi}{30}$.

(c) $\frac{\pi}{40}$.

(d) $\frac{\pi}{60}$.

$$\frac{\pi}{4}\int (x - x^2)^2 = \frac{\pi}{4}\int x^4 - 2x^3 + x^2$$

$$\frac{\pi}{4}\left(\frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3}\right)$$

$$\frac{\pi}{4}\left(\frac{1}{5} - \frac{2}{4} + \frac{1}{3}\right)$$

$$\frac{\pi}{4}\left(\frac{12}{60} - \frac{30}{60} + \frac{20}{60}\right)$$

$$\frac{\pi}{120}$$

8. Solve the differential equation: $y' = \cot x \ln(sinx)$; $y(\frac{\pi}{2}) = 0$

(a) $\frac{(\ln(\cos x))^2}{2} + 2$.

(b) $\frac{(\ln(\sin x))^2}{2}$.

(c) $\ln(\sin x)$.

(d) $\frac{(\ln(\sin x))^2}{2} + 1$.

$u = \sin x$

$du = \cos x dx$

$dx = \frac{du}{\cos x}$

$\frac{d}{dx} e^{-x} = -\frac{e^{-x}}{5}$

Question 5. *(28%) Evaluate:*

1. $\int \frac{e^{2x}}{e^x+1} dx$

$\Rightarrow \int \frac{e^{2x}}{e^x(1+\frac{1}{e^x})} \Rightarrow \int \frac{e^x}{1+\frac{1}{e^x}} \Rightarrow -\int \frac{-e^{-x}}{1+e^{-x}} = -\ln|1+e^{-x}|$

$u$ $\frac{u}{u+1}$

$u =$

$\frac{1}{\sqrt{x^2+1}}$

2. $\int \frac{x^2+1}{x^2-x} dx$

$\frac{x^2 - x}{1+x}$

$\Rightarrow \int 1 + \frac{1+x}{x^2-x} dx \Rightarrow \int 1\, dx + \int \frac{1+x}{x(x-1)} dx$

$\int \frac{1+x}{x(x-1)} dx = \int \frac{A}{x} dx + \int \frac{B}{(x-1)} dx$ $\qquad -1 + B = 1$

$\Rightarrow 1+x = Ax - A + Bx \Rightarrow Ax + Bx = x \to ①$

$-A = 1 \to ②$

$\Rightarrow A = -1 \;\&\; B = 2$

$\Rightarrow \int \frac{-1}{x} dx + \int \frac{2}{x-1} dx \Rightarrow -\ln|x| + 2\ln|x-1|$

$\Rightarrow$ Final answer: $x + -\ln|x| + 2\ln|x-1|$

3. $\int_{-1}^{1} \frac{dx}{\sqrt{1-x}}$

$\int \frac{dx}{\sqrt{1-x}} \Rightarrow u = 1-x,\; du = -dx \qquad \Rightarrow \int \frac{dx}{\sqrt{1-x}} \Rightarrow u = 1-x,\; du = -dx$

$\Rightarrow \int \frac{-du}{\sqrt{u}} \Rightarrow -2\sqrt{u}\Big|_{-b}^{1} \qquad \Rightarrow -2\sqrt{u}$

$\Rightarrow -2 + 2\sqrt{b} \qquad \int_{-1}^{0} \frac{dx}{\sqrt{1-x}} + \int_{0}^{1} \frac{dx}{\sqrt{1-x}}$

$\lim_{b \to -1^+} -2 + 2\sqrt{b} =$

$\Rightarrow \lim -2\sqrt{u}\Big|_{-1}^{0} + -2\sqrt{u}\Big|_{0}^{1}$

$\Rightarrow +2\sqrt{b} + -2$

$\lim_{b \to -1^+} +2\sqrt{b} + -2 = \text{diverge}$ $\quad$ D.N.E $\quad$ 2

Birzeit University-Mathematics Department
Calculus II-Math 132

First Exam
Second Semester 2013/2014

Name(Arabic):...[redacted]...
Number:[redacted]

Instructor of Discussion(Arabic):...[illegible]...
Section:...7...

18

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**Question 1.**(19%) Circle the correct answer:

12

(1) Let $y = x^{\tan^{-1} x}$, then $y' =$

(a) $\tan^{-1} x \ln x$.

(b) $x^{\tan^{-1} x}\left(\frac{\ln x}{1+x^2} + \frac{\tan^{-1} x}{x}\right)$.

(c) $x^{\tan^{-1} x}\left((1+x^2)\ln x + (\tan^{-1} x)\ln x\right)$.

(d) $x^{\tan^{-1} x}\left(\frac{x}{1+x^2} + \tan^{-1} x\right)$.

$\ln y = \ln x^{\tan^{-1} x}$

$\ln y = \tan^{-1} x \ln x$

$\frac{1}{y}\frac{dy}{dx} = \tan^{-1} x \frac{1}{x} + \ln x \frac{1}{1+x^2}$

$y' = \left(\frac{\tan^{-1} x}{x} + \frac{\ln x}{1+x^2}\right) x^{\tan^{-1} x}$

(2) $\int_2^4 \frac{dx}{x\sqrt{x^2-4}} =$

(a) $\frac{\pi}{4}$.

(b) $\frac{\pi}{2}$.

(c) $\frac{\pi}{6}$.

(d) $\frac{\pi}{3}$.

$\frac{1}{2}\sec^{-1}\frac{x}{2}$

$x^2 - 4 = u$

(3) $\int_2^4 \operatorname{sech}(\ln x)dx =$

(a) $\tanh(4) - \tanh(2)$.

(b) $\ln 4 - \ln 2$.

(c) $\ln 17 - \ln 4$.

(d) $\ln 17 - \ln 5$.

$\ln x = u$

$du = \frac{1}{x}dx$

$x = e^u$

$\int e^u \operatorname{sech} u \, du$

(4) $\lim_{x\to\infty}(x - \ln x) =$

(a) 0.

(b) $\infty$.

(c) 1.

(d) Does not exist.

(5) $\int_0^1 \frac{dx}{\sqrt{3+2x-x^2}} =$

(a) $\frac{\pi}{3}$.

(b) $\frac{\pi}{6}$.

(c) $-\frac{\pi}{6}$.

(d) $\frac{\pi}{4}$.

$-(x^2-2x-3)$

$-(x^2-2x-3)$

$(x^2-2x+1)$

$-((x-1)^2-4)$

$-(x-1)^2+4$

$(x-1)=u$

$du=dx$

$\int \frac{du}{\sqrt{2^2-u^2}}$

$\sin^{-1}\frac{u}{2}+c$

$\sin^{-1}\frac{x-1}{2}$

(6) $\int_1^e \frac{2^{\ln x}}{x}dx =$

(a) 1.

(b) $\frac{1}{\ln 2}$.

(c) $-\ln 2$ .

(d) $\frac{2}{\ln 2}$.

$\ln x = u$

$du = \frac{1}{x}dx$

$2^u \Rightarrow e^{u\ln 2}$

$\int 2^u du$

$\frac{2^u}{\ln 2}$

$\left(\frac{2}{\ln 2}-\frac{1}{\ln 2}\right)$

$\frac{1}{\ln 2}$

(7) $4^{\log_2(4)} =$

(a) 2.

(b) 4.

(c) 8.

(d) 16.

$\Rightarrow 2^{\log_2 4^2} = 4^2 = 16$

(8) Let $f(x) = 2x + e^x$ then $(f^{-1})'(1) =$

(a) 1.

(b) $\frac{1}{2}$.

(c) $\frac{1}{3}$.

(d) $\frac{1}{2+e}$.

$y = 2x + e^x$

$f' = 2 + e^x$

$f' = 2 + \frac{1}{2x}$

$\frac{1}{2+\frac{1}{2x}}$

(9) $\int_0^{\pi/3}(\sec\theta)^4 d\theta =$

(a) 3.

(b) $\sqrt{3}$.

(c) $3\sqrt{3}$.

(d) $2\sqrt{3}$.

$\sec^2\theta\,(1+\tan^2\theta)\,d\theta$

$\tan\theta = u$

$du = \sec^2\theta\, d\theta$

$\int (1+u^2)\,du$

$u + \frac{u^3}{3} = \tan\theta + \frac{\tan^3\theta}{3}\Big|_0^{\pi/3}$

$\sqrt{3} + \frac{1}{3}(\sqrt{3})^3 - 0$

(10) The half-life of a radioactive element is 3500 years. The decay rate of the element is

(a) $3500 \ln 2$.

(b) $\frac{3500}{\ln 2}$.

(c) $\frac{\ln 2}{3500}$.

(d) $\ln 2$.

$t = \frac{\ln 2}{K}$

$K = \frac{\ln 2}{t} = \frac{\ln 2}{3500}$

(11) $\int_0^{\pi/3} \cos^3 x dx =$

(a) $3\sqrt{3}$.

(b) $\frac{\sqrt{3}}{8}$.

(c) $\frac{3\sqrt{3}}{8}$.

(d) $\frac{\sqrt{3}}{2}$.

$\cos x(1 - \sin^2 x)\, dx$

$\sin x = u$

$du = \cos x \, dx$

$\int (1 - u^2)\, du$

$u - \frac{u^3}{3}$

$\sin x - \frac{\sin^3 x}{3} \Big|_0^{\frac{\pi}{3}}$

$\frac{\sqrt{3}}{2} - \frac{\sqrt{3}^3}{8} - 0$

$\frac{\sqrt{3}}{2} - \frac{\cancel{3}\sqrt{3}}{8 \cdot \cancel{3}}$

$\frac{4\sqrt{3}}{8} - \frac{\sqrt{3}}{8} = \frac{3\sqrt{3}}{8}$

(12) $\cos^{-1}(\cos(-\frac{\pi}{2})) =$

(a) $-\frac{\pi}{2}$.

(b) 0.

(c) $\frac{\pi}{2}$.

(d) None.

$0 \leq \cos^{-1} \leq \pi$

(13) The following functions grow from slowest to fastest

(a) $2^x, x^{10}, x^x, (\ln x)^x$.

(b) $(\ln x)^x, x^x, 2^x, x^{10}$.

(c) $x^{10}, 2^x, (\ln x)^x, x^x$.

(d) $x^{10}, 2^x, x^x, (\ln x)^x$.

(14) The range of the function $\cos(\sin^{-1} x)$ is

(a) ~~$[0, 1]$~~.

(b) $[-1, 1]$.

(c) $[-1, 0]$.

(d) $[0, \pi]$.

$\sin^{-1}$

$-\frac{\pi}{2}, \frac{\pi}{2}$

$\frac{\pi}{2}$

$-\frac{\pi}{2}$

(15) Let $f(x) = \frac{x}{1-x}$, $x \neq 1$ then $f^{-1}(x) =$

(a) $\frac{x}{1-x}$.

(b) $\frac{1-x}{x}$.

(c) $\frac{x}{x+1}$.

(d) $\frac{x+1}{x}$.

$y = \frac{x}{1+x}$

$y - yx = x$

$y = x + yx$

$x(1+y) = y$

$x =$

$y =$

(16) $\sin(\tan^{-1} x) =$

(a) $\frac{x}{\sqrt{1+x^2}}$.

(b) $\frac{1}{\sqrt{1+x^2}}$.

(c) $\frac{1}{\sqrt{1-x^2}}$.

(d) $\frac{x}{\sqrt{1-x^2}}$.

$u =$

$\sin(\cos^{-1}(x)) =$

~~$x = 1+y$~~

$x(1+y) = y$

$x = \frac{y}{1+y}$

$y = \frac{x}{1+x}$

(17) Let $y = \log_2(\ln x^2)$ then $y' =$

(a) $\frac{1}{x \ln x}$.

(b) $\frac{1}{(\ln 2) x \ln x}$.

(c) $\frac{2}{(\ln 2) x \ln x}$.

(d) $\frac{1}{x^2 \ln(x^2)}$.

$\frac{\ln(\ln x^2)}{\ln 2}$

~~$\frac{\ln(2\ln x)}{\ln 2}$ $\frac{1}{x \ln 2} \cdot \frac{1}{2x \ln x}$~~

~~$\frac{2}{x \ln 2 \ln x}$~~

$2 \frac{\ln(\ln x)}{\ln 2}$

$\frac{2}{\ln 2} \frac{1}{x \ln x}$

(18) $\lim\limits_{x \to 0+} \left(\frac{1}{x}\right)^x =$

(a) 1.

(b) 0.

(c) $\infty$.

(d) Does not exist.

$\ln\left(\frac{1}{x}\right)^x$

$x \ln \frac{1}{x}$

$\frac{\ln \frac{1}{x}}{\frac{1}{x}} = \frac{0}{0}$ $\frac{-\ln x}{\frac{1}{x}} =$

$\frac{+\frac{1}{x}}{+\frac{1}{x}}$ $\frac{1}{x} =$

(19) $\int_0^1 x \tan^{-1} x dx =$

(a) $\pi - 1$.

(b) $\frac{\pi}{4} - 1$.

(c) $\frac{\pi}{4} - \frac{1}{2}$.

(d) $\frac{\pi}{2} - 1$.

$u = \tan^{-1} x$ $du = \frac{1}{1+x^2} dx$

$dv = x dx$ $dv = \frac{x^2}{2}$

$\frac{x^2}{2} \tan^{-1} x - \frac{1}{2}\int \frac{x^2}{1+x^2} dx$

$x^2 = u - 1$

$1 + x^2 = u$

$du = 2x dx$

$\frac{u-1}{\ }$ $du$



$\frac{(e^x - e^{-x})}{(e^x + e^{-x})}$

Question 2(6%) Use a substitution then integrate by parts to solve the integral

$$\int \sin(\ln x)dx$$

$z = \ln x \Rightarrow x = e^z$

$dz = \frac{1}{x} dx$

$\int e^z \sin z \, dz$

$u = e^z \qquad du = e^z dz$

$dv = \sin z dz \qquad v = -\cos z$

$-e^z \cos z + \int e^z \cos z \, dz$

$w = e^z \qquad dw = e^z dz$

$ds = \cos z dz \qquad s = \sin z$.

$e^z \sin z - \int \sin z \, e^z dz$

$\int e^z \sin z \, dz = -e^z \cos z + e^z \sin z - \int \sin z \, e^z dz$

6

$\int e^z \sin z \, dz = \frac{1}{2}\left[-e^z \cos z + e^z \sin z\right]$

$= \frac{1}{2}\left[-e^{\ln x} \cos \ln x + e^{\ln x} \sin \ln x\right]$

5

$= \frac{1}{2}\left[-x \cos \ln x + x \sin \ln x\right]$

$\frac{x}{2}\left[\sin \ln x - \cos \ln x\right] + C$

BIRZEIT UNIVERSITY

$\frac{84}{100}$

**Mathematics Department**

**First Hour Exam /summer 2015**

**Math 132**

**Student name:** Moath Raddad **Section** 2 **Student no.:** 1141127

$\frac{25}{16} - \sinh^2 = 1$

$\sinh^2 x = \frac{25}{16} - \frac{16}{16} = \frac{9}{16}$
$= \frac{3}{4}$

**Q#1 80% Circle the correct answer.**

1. If $\cosh x = \frac{5}{4}$ , $x>0$ then $\sinh x =$

a) $\frac{3}{5}$ b) $\frac{3}{4}$ c) $\frac{4}{5}$ d) $\frac{5}{4}$

2. The order of the functions $x^x$ , $e^x$ , $\sqrt{x}$ , $\log x$ from slower growing to fastest growing as $x \to \infty$

a) $x^x$ , $e^x$ , $\sqrt{x}$ , $\log x$ b) $\sqrt{x}$ , $\log x$ , $x^x$ , $e^x$
c) $\log x$ , $\sqrt{x}$ , $x^x$ , $e^x$ d) $\log x, \sqrt{x}$ , $e^x$ , $x^x$

$\lim_{x \to \infty} \frac{x^x}{e^x}$

$\left(\frac{x}{e}\right)^x$

64

3. $\int_{\ln 2}^{\ln 4} \frac{e^x}{e^x+1} dx =$

a) $\ln 2$ b) $\ln 3$ c) $\ln 4$ d) $\ln 5 - \ln 3$

$f(x) = \ln \frac{x^x}{e^x}$

$\frac{e^x \cdot e^{-x}}{(e^x+1)\, e^{-x}}$

$\frac{1}{1+e^{-1}}$

$\int_{\ln 2}^{\ln 4} \ln|e^x+1|$

$\frac{1}{x}$

$\left[\ln e^{\ln 4}+1\right] - \left[\ln e^{\ln 2}+1\right]$

$\ln 4 + 1 - \ln 2 - 1$

$\ln 4 - \ln 2$

$\ln \frac{4}{2} = \ln 2$

4. if $y = (\ln x)^x$ then $\frac{dy}{dx}$

a) $(\ln x)^x[\frac{1}{\ln x} + \ln(\ln x)]$ b) $(\ln x)^{x-1}$

c) $(\ln x)^{x-1}$ d) $(\ln x)^x(\frac{2\ln x}{x})$

$\ln y = \ln(\ln x)^x$

$\frac{y'}{y} = x \ln(\ln x)$

$x \cdot \frac{\frac{1}{x}}{\ln x} + \ln(\ln x) \cdot 1$

$\left[\frac{1}{\ln x} + \ln(\ln x)\right] (\ln x)^x$

1

$\ln(\frac{1}{\sqrt{3}}) \quad \frac{1}{3}x^{\frac{3}{3}} - \frac{4}{3}$

$-\ln 2$

$\ln 4 - \ln 3 - \ln 2$

$$5.\int_{1}^{\frac{1}{\sqrt{3}}} \frac{dx}{(1+x^2)\tan^{-1}x} =$$

a) $\ln 4 - \ln 3$ b) $\ln 2 - \ln 3$

c) $2\ln\sqrt{3}$ d) $\frac{\pi}{12}$

$\tan^{-1}$ $\quad 1 - x^2 \quad x$

$\ln|u|$

$\ln\frac{\pi}{8} - \ln\frac{\pi}{4} \qquad \ln\frac{4}{6}$

$\frac{du}{u\sqrt{u^2-1}} =$

$(x^2-2x+1)-1$

$$6.\int \frac{dx}{(x-1)\sqrt{x^2-2x}}$$

a) $\sqrt{x^2-x}+c$ b) $\frac{1}{2}\ln|x^2-2x|+c$

c) $\sec^{-1}|x-1|+c$ d) $\sinh^{-1}(x-1)+c$

$(x-1)(x-1)$

$\sqrt{(x-1)^2-1}$

$u = x-1$

$du = dx$

$\sec^{-1}(u$

$\int \frac{du}{u\sqrt{u^2-1}}$

$\sec^{-1}|u^2-1|$

$$7.\int_{-1}^{0} \frac{dx}{x^2+2x+2} =$$

a) 1 b) $\frac{1}{2}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

$(x-1)^2$

$x^2-2x$

$\frac{1}{\cancel{(x+1)(x+1)+2=1}}$

$$8.\int x^2e^{3x}dx =$$

a) $\frac{e^{3x}}{3}(9x^2+6x+2)+c$ b) $e^{3x}(\frac{1}{3}x^2-\frac{2}{9}x+\frac{2}{27})+c$

c) $\frac{e^{3x}}{27}(9x^2+6x-2)+c$ d) $\frac{e^{3x}}{3}(9x^2-3x+2)+c$

$x^2 \quad e^{3x}$

$2x \quad \frac{e^{3x}}{3}$

$2 \quad \frac{e^{3x}}{9}$

$0 \quad \frac{e^{3x}}{27}$

$\frac{1}{(x^2+2x+1)+1}$

$(x+1)(x+1)$

$(x+1)^2+1$

$e^{3x}(\frac{1}{3}x^2 - \frac{2}{9}x + \frac{2}{}$

$x^2 \cdot \frac{e^{3x}}{3} + 2x \cdot \frac{e^{3x}}{9} + \frac{2e^{3x}}{27}$

$u = x+1$

$du = dx$

$$9.\lim_{x\to 0}\frac{\sinh\ 2x}{\sin\ x} =$$

a) 2 b) 1 c) $\frac{1}{2}$ d) doesnt exist

$\int \frac{1}{u^2+1}$

$\frac{\cos}{0}$

$\frac{e^x + e^{-x}}{2} \cdot 2$

$\frac{\cosh 2x \cdot 2}{\cos x}$

$\tan^{-1}|u|$

$\tan^{-1}(x+1)$

$e^0 - e^0 = 1 - 1 = 0$

$\tan(1) - \tan(0)$

$\frac{\pi}{4} - 0$

$e^x + 3x + 5 = 6$

$e^x + 3x = 1$

$\boxed{x = 0}$

$(f^{-1})'(6) = \frac{1}{f'(f^{-1}(6))}$

$\frac{1}{f'(0)}$

$e^x + 3 = 1 + 3 \quad \frac{1}{4}$

10. If $f(x) = e^x + 3x + 5$ then $(f^{-1})'(6) =$

a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{1}{e+3}$ d) 3

$4 \times 4 \times 4 \times 4$

$\frac{2\times 2}{4} \times \frac{2\times 2}{4} \times \frac{2\times 2}{4}$

$\frac{\ln 32}{\ln 4}$

$\frac{\ln 2^5}{\ln 2^2} \quad \frac{5}{2}$

11. $\log_4 32$

a) 2 b) $\frac{1}{2}$ c) $\frac{5}{4}$ d) $\frac{5}{2}$

$f(x) = 6 \quad x = 0$

$6 = e^x + 3x + 5$

$\ln y = \sin x \ln 2$

$\frac{y'}{y} = \ln 2 \cos x \cdot y$

$y' = \ln \cos \pi \cdot 2^{\sin \pi}$

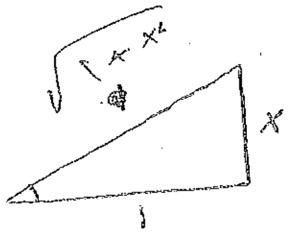
$\ln 2 \cdot 2^{\sin \pi} \quad 2^0 = 1$

12. if $y = 2^{\sin x}$ then $\frac{dy}{dx}$ when $x = \pi$ is:

a) 0 b) 1 c) $\ln 2$ d) $-\ln 2$

13. if $y = \tan^{-1}(\frac{1}{x})$ then $\sin y =$

a) $\frac{1}{\sqrt{1+x^2}}$ b) $\frac{\sqrt{1+x^2}}{x}$ c) $\frac{x}{\sqrt{1+x^2}}$ d) $\frac{1}{\sqrt{1-x^2}}$



14. $\int_0^1 \frac{1}{\sqrt{x} + x\sqrt{x}} dx$

a) $\pi$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

$\frac{1}{\sqrt{x} + x\sqrt{x}}$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} \quad du\, 2\sqrt{x} = dx$

$\frac{du \cdot 2u^2}{u + u^2 \cdot u}$

$\frac{1}{u + xu}$

$2 \int \frac{u^2}{u + u^3}$

$du \cdot 2\sqrt{x} = dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} \Rightarrow du \cdot 2\sqrt{x} = dx$

$\frac{\cosh u}{\cancel{u}} \cdot \cancel{2u} \, du$

$2 \cosh u \cdot u$

15. $\int_0^1 \frac{\cosh(\sqrt{x})}{\sqrt{x}} dx =$

a) $\frac{(e-1)^2}{2}$ b) $2\sinh 1$ c) $2(\cosh 1) - 2$ d) $\frac{e}{2}$

$u \quad \cosh u$

$0 \quad \sinh u$

$2 \cdot u \sinh u$

$2\sqrt{1} \sinh\sqrt{1} - 20$

$2 \sinh$

16. $\int \frac{dx}{\sqrt{2x - x^2}} =$

a) $2\sqrt{2x - x^2} + c$ b) $\sin^{-1}(\frac{x-1}{2}) + c$

c) $\sin^{-1}(x-1) + c$ d) $\sec^{-1}(x-1) + c$

$-(x^2 - 2x + 1 - 1)$

$-((x-1)^2 + 1)$

$1 - (x-1)^2$

17. $\int \tan^{-1} x \; dx =$

a) $x \tan^{-1} x + \sqrt{1+x^2} + c$ b) $x \tan^{-1} x - \ln\sqrt{1+x^2} + c$

c) $x \tan^{-1} x - \sqrt{1+x^2} + c$ d) $x \tan^{-1} x - 2\sqrt{1+x^2} + c$

$\frac{1}{\sqrt{1-u^2}}$

$x = a \tan\theta$

$\frac{x}{a} = \tan\theta$

$\tan^{-1}$

18. If $f(x) = \frac{x+1}{x-2}$ then

a) $f^{-1}(x) = \frac{2x+1}{x-1}$ b) $f^{-1}(x) = \frac{x-1}{x+2}$

c) $f^{-1}(x) = \frac{2x+1}{x+2}$ d) $f^{-1}(x) = \frac{2x-1}{x+2}$

$y(x-2) = (x+1)$

$f^{-1}(x) \cdot f(\frac{1}{x})$

$2 - x$

19. $4^{\ln 2} =$

a) 2 b) 4

c) $2\ln 2$ d) $\ln 4$

$2^{2\ln 2} = 2^{\ln 4}$

$f(f^{-1}(x)) = x$

20. $\int_1^{\sqrt{e}} \frac{dx}{x\sqrt{1-(\ln x)^2}} ==$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$

c) $\frac{\pi}{2}$ d) $\pi$

$\sec^{-1}(\ln x) - \sec^{-1}(1)$

$\sec^{-1}(\frac{1}{2})$

$x + 1 - 2 + 2$

$3 + 2y - 2$

$\frac{2y+1}{y-1}$

$\frac{x-2}{x-2} + \frac{3}{x-2}$

$y - 1 = \frac{3}{x-2}$

$x - 2 = \frac{3}{y-1} + \frac{2(y-1)}{y-1}$

4

$y = 1 + \frac{3}{x-2}$

$\frac{1}{\cos}$

$y - 1 = x - 2$

$\frac{\pi}{6} - \frac{\pi^3}{3x^2} \quad 4 + 1$

$\frac{1}{\cos 0}$

$2\frac{\pi}{6} = \frac{\pi}{3}$

Q#2 20% Evaluate.

1. $\int \ln(x^2+1)\,dx$

$u = \ln(x^2+1) \qquad dv = dx$

$du = \frac{2x}{x^2+1} \qquad v = x$

~~u = ln~~

$u = x^2+1$

$\frac{du}{2x} = \frac{2x\,dx}{2x}$

$\int \frac{\cancel{\ln u}\,du}{2x}$

$x\ln|x^2+1| - \int \frac{2x^2}{x^2+1}$

$\cancel{x\ln|x^2+1| - 2\int \frac{x^2}{x^2+1}}$

$u = x^2+1$

$du = 2x$

$1 - 1$

$\cancel{\int \frac{2x^2}{u} \cdot \frac{du}{2x}} \qquad \cancel{\frac{2}{1+x^{-2}}}$

$\cancel{x\ln|x^2+1| - \int \frac{2x^2}{x^2+1}}$

~~تابع خلف الورقة~~

$u = 1 - x$

$du = -2x$

$\frac{\sqrt{u}}{-2x\, x^2}$

$(1-x)(1+x)$

$\sqrt{1-x^2} \cdot \frac{\sqrt{1+x^2}}{1+\sqrt{x}}$

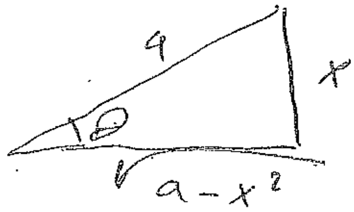
$\frac{1-x^2}{}$

$$2.\int \frac{\sqrt{1-x^2}}{x^2}dx$$

~~$a = \sin$~~

$x = a \sin\theta$

$x^2 = a^2 \sin^2\theta$




$a \cos\theta = \frac{\sqrt{a - x^2}}{a}$

$dx = \cos\theta\, d\theta$

$\int \frac{\cos\theta}{\sin^2\theta} \cdot \cos\theta\, d\theta$

$\int \frac{\cos^2\theta}{\sin^2\theta}$

$\int \cot^2\theta\, d\theta$

$\int \csc^2\theta - 1\, d\theta$

$-\cot\theta - \theta + C$

$-\frac{\sqrt{a-x^2}}{x} - \sin^{-1}(x) + C$

$\tan^2$

$(\sec^2 - 1$

$\tan -$

$\frac{1}{\tan}$

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BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT
MATH132 -Test One-
Fall 2015/2016

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• Name.................................... • Number.................... • Section....................

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**Question 1.** *(10 points)* Circle the best answer or write your answer in the designated area (in each case, show your work).

1. For $x > 0$, $\int (\frac{1}{2x} \int_1^x \frac{du}{u}) dx =$

   a) $\frac{1}{x^3} + C$

   b) $\ln(\ln x) + C$

   c) $\frac{(\ln x)^2}{4} + C$

   d) $\frac{\ln(x^2)}{4} + C$

2. If $f(x) = (x^2+1)^{(2-3x)}$. Then $f'(1) =$

   a) $-\ln(8e)$

   b) $-\frac{1}{2}\ln(8e)$

   c) $-\frac{3}{2}\ln(2)$

   d) $-\frac{1}{2}$

3. $\sec^{-1} 4 + \sin^{-1} \frac{1}{4} =$

   a) $\frac{\pi}{4}$.

   b) $-\frac{\pi}{2}$.

   c) $\frac{\pi}{2}$.

   d) $-\frac{\pi}{4}$.

4. $\lim_{x \to \infty} x^{\frac{1}{\sqrt{x}}}$

   a) 1

   b) 0

   c) $-1$

   d) $\infty$

5. Find $\lim_{x \to 2} \dfrac{\int_2^x \cos t dt}{x^2 - 4}$ and write your answer below.

Your answer

6. Evaluate $\int_0^{\frac{\sqrt{3}}{2}} \dfrac{1 + x^3}{\sqrt{1 - x^2}}\, dx$ and write your answer below.

Your answer

7. A puppy weighs 2 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during it's first 6 months is increasing at a rate proportional to it's weight, then how much will the puppy weigh when it's 3 months old .

a) 4.6 pounds.

b) 5.6 pounds.

c) 6.5 pounds.

d) 7.5 pounds.

8. $\int_0^{\frac{\pi}{4}} \dfrac{2e^{\tan x}}{\cos^2 x}\, dx =$

a) 2

b) $2e^{-1}$

c) $2e - 2$

d) $2e + 2$

9. Which of the following functions grow faster than $\ln x$ as $x \to \infty$ :

a) $\dfrac{1}{x}$.

b) $\ln \sqrt{x}$.